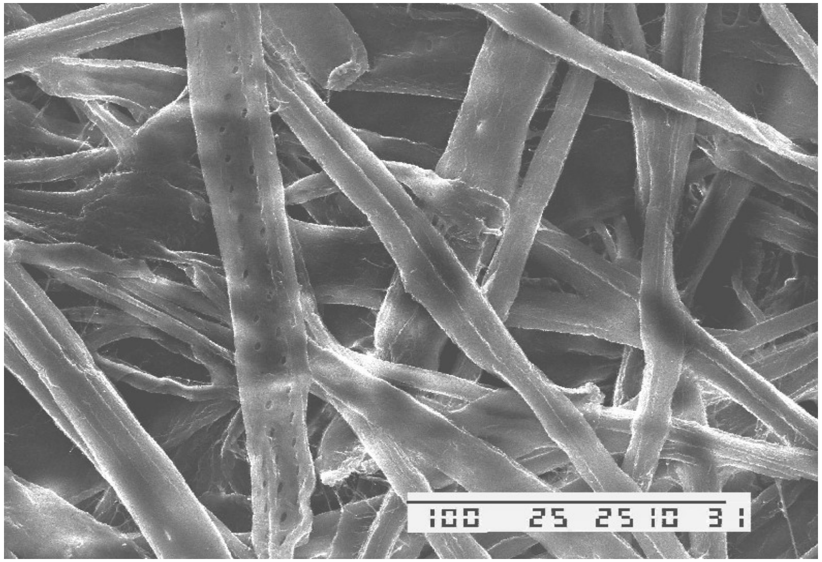
Applicability of Engineering Models in Multiscale Modeling of Natural Fiber Hygro-Elastic Properties



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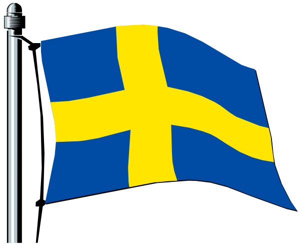
Where is Luleå?

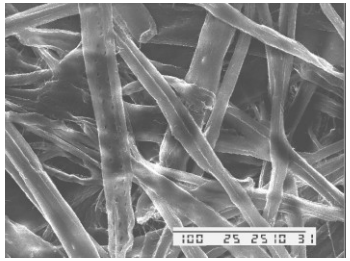


Outline

- Multiple length scales in natural fibers
- Suitable models for each scale
- Homogenized properties (stiffness and moisture expansion) on each scale
- Effect of different parameters
- Composite properties (stiffness and moisture expansion)
- Conclusions

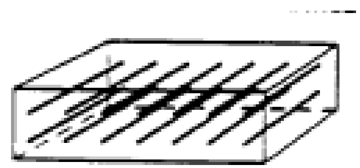
Multiple length scales

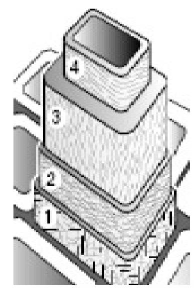
Composite or mat with fiber orientation



- Elastic properties
- Moisture expansion

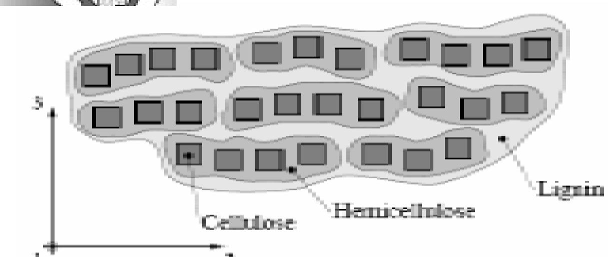
Aligned fiber composite (wood)





Fiber wall

S1,S2 etc layers



Lignin, Hemicellulose, Cellulose

Objective

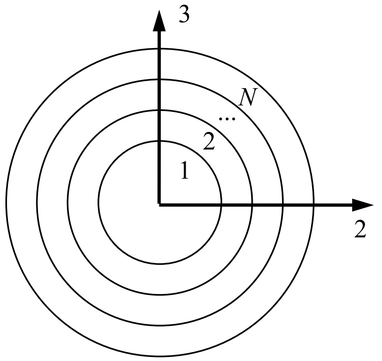
- To analyze stiffness and moisture expansion of natural fibers and their composites

Approach

- Analyze suitable models for linking different scales
- Perform multiscale analysis

Suitable models

CCA= concentric cylinder assembly



Orthotropic materials in local axes

Composite with fiber orientation

Laminate analogy

Fibers and aligned composite (wood)

CCA

Fiber wall

Laminate theory

S1,S2 etc layers

CCA, rule of mixtures

Lignin,

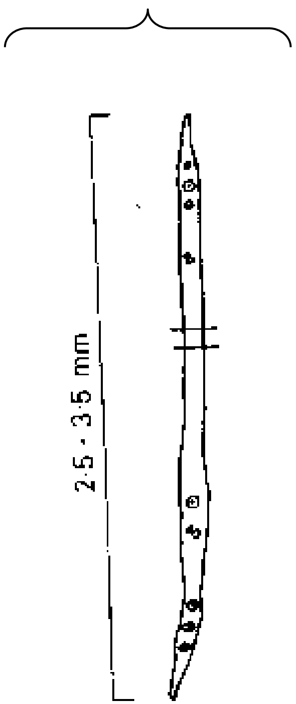
Hemicellulose

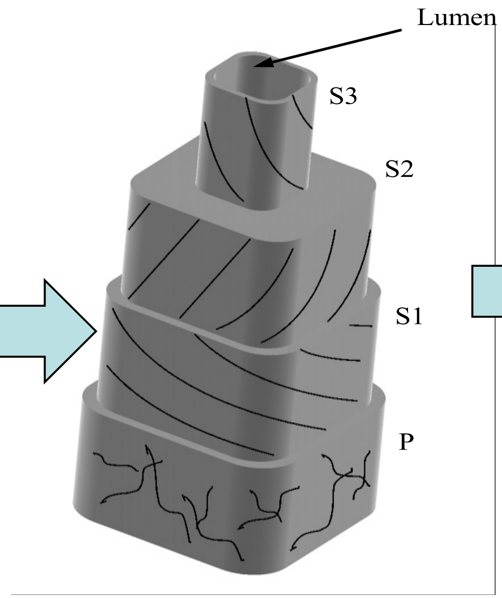
Cellulose

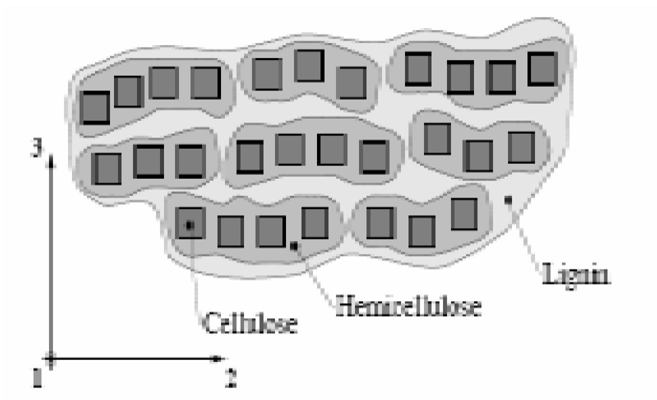
- Elastic properties
- Moisture expansion

Ultrastructure of the fiber wall

Softwood fiber



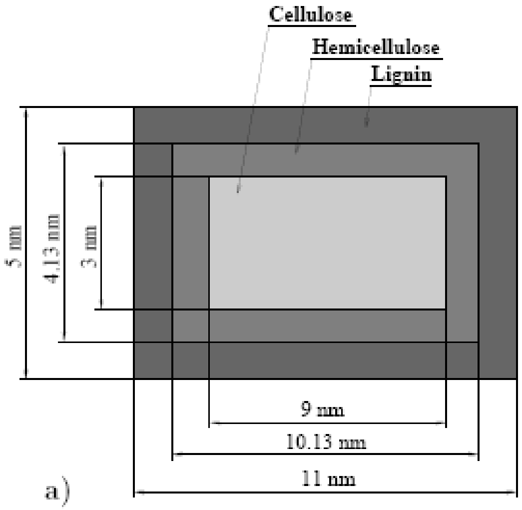




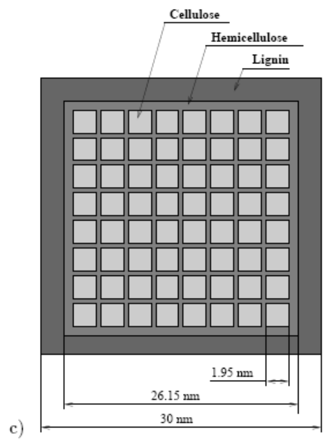
Long cylinder structures

(L/d ≈ 100)

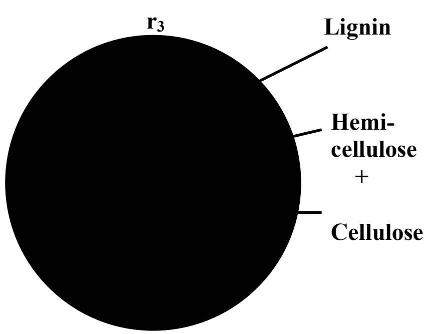
Layer properties: unit cell



Constituent		E_1^k (GPa)	E_2^k (GPa)	G_{12}^k (GPa)	ν_{12}	ν_{23}
1	Cellulose	150.0	17.5	4.5	0.08571	0.50
2	Hemicellulose	16.0	3.5	1.5	0.4571	0.4
3	Lignin	2.75	2.75	1.034	0.33	0.33



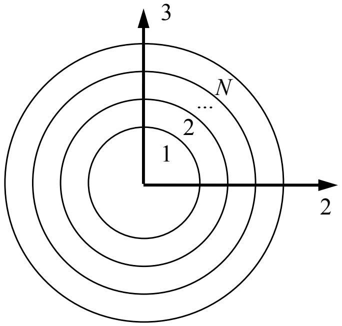
Model B



Volume fractions are different in different cell wall layers

The Concentric Cylinder Assembly

An *N*-phase cylinder assembly with orthotropic constituents



- Calculates all engineering constants of the homogenized assembly
- Suitable for natural fibers (having layered structure)

”Rule of mixtures” models

Modulus in “axial” direction

$$E_1 = \sum_{k=1}^{3} V_k E_1^k = V_C E_1^C + V_H E_1^H + V_L E_1^L$$

Poisson’s ratio

$$\nu_{12} = \sum_{k=1}^{3} V_k \nu_{12}^k = V_C \nu_{12}^C + V_H \nu_{12}^H + V_L \nu_{12}^L$$

Transverse modulus and shear modulus

(Halpin-Tsai with two step homogenization)

C+H model: cellulose in hemicellulose matrix

(C+H)+L model: C+H in lignin matrix

Moisture swelling in natural fiber composites

Definition

$$\beta_{k}^{m} = \frac{\varepsilon_{k}^{m}}{\Delta m} \qquad\qquad \sigma_{i} = C_{ij}(\varepsilon_{j} - \varepsilon_{j}^{m})$$

ε_{k}^{m} is free swelling strain in $k-$direction due to moisture weight content change by Δm

CCA model has been applied on cell wall layer and also on fiber scale

Average moisture content

$$\Delta m_{assembly} = \sum_{k} \Delta m_{k} \frac{\rho_{k}}{\rho_{assembly}} V_{k}$$

$$\rho_{assembly} = \sum_{k} \rho_{k} V_{k}$$

Input:

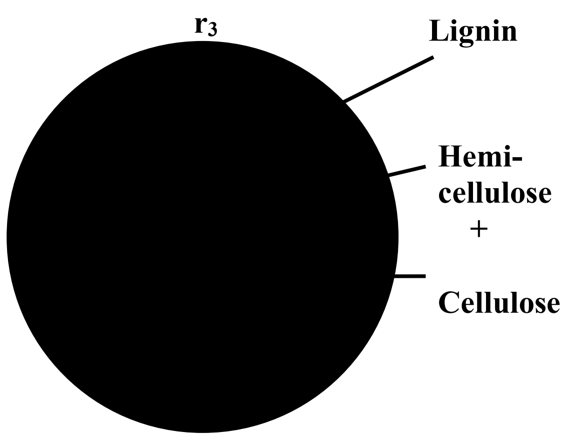
moisture contents,

expansion coefficients

in constituents

S2 Layer properties

Model B

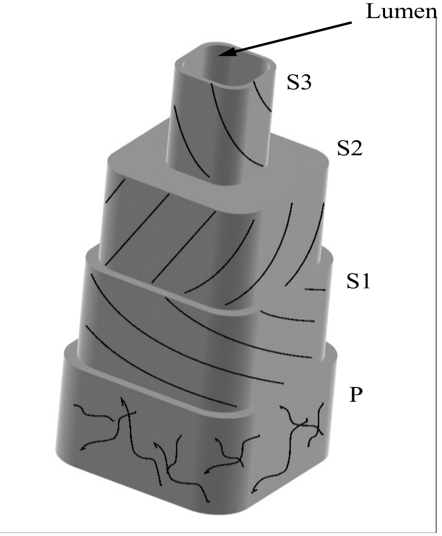


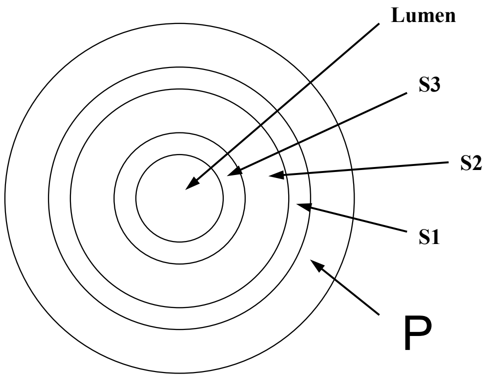
Property	Model B	ROM models
E_1 (GPa)	72.278	72.169
E_2 (GPa)	5.964	6.090
ν_{12}	0.243	0.262
G_{12} (GPa)	2.112	2.112
G_{23} (GPa)	2.056	2.260
ν_{23}	0.451	0.347

According to CCA the parameter in H-T relationships for transverse modulus is different

$$\xi \approx .2$$

Fiber wall=laminate **(symmetric!!!)**





Stiffness in local axes $Q_{11}^k = \frac{E_1^k}{1-\nu_{12}\nu_{21}}$

In global axes $m = \cos\theta$

$$\bar{Q}_{11}^k = Q_{11}^k m^4 + Q_{22}^k n^4 + Q_{12}^k 2m^2 n^2 + Q_{66}^k 4m^2 n^2$$

Laminate stiffness matrix

$$A_{ij} = \sum_k \bar{Q}_{ij}^k h_k$$

Laminate compliance matrix

$$[S] = h[A]^{-1}$$

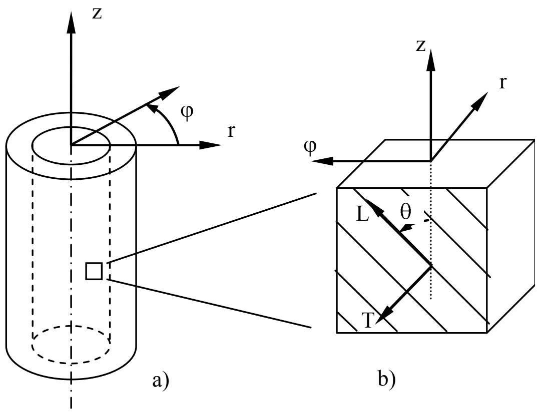
Cell wall elastic constants

$$E_z = \frac{1}{S_{11}} \qquad E_\varphi = \frac{1}{S_{22}}$$

	Earlywood		Latewood	
	Thickness	Angle	Thickness	Angle
P	0.1	Random	0.1	Random
S1	0.2	50-70	0.3	50-70
S2	1.4	10-40	4.0	0-30
S3	0.03	60-90	0.04	60-90

Influence of Helical Fiber Structure

The axial deformation is coupled with torsion



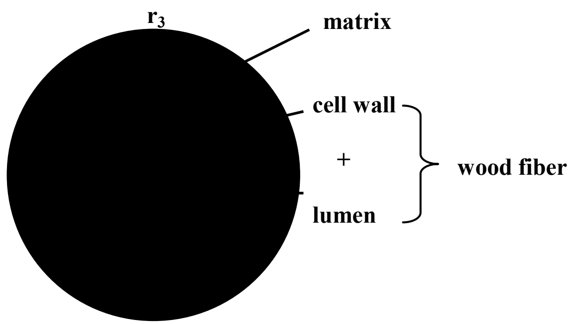
$$
\begin{Bmatrix} \sigma_z \\ \sigma_\varphi \\ \sigma_r \\ \sigma_{\varphi r} \\ \sigma_{zr} \\ \sigma_{z\varphi} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26} \\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36} \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{36} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_z \\ \varepsilon_\varphi \\ \varepsilon_r \\ \gamma_{\varphi r} \\ \gamma_{zr} \\ \gamma_{z\varphi} \end{Bmatrix}
$$

Using orthotropic fiber wall models:

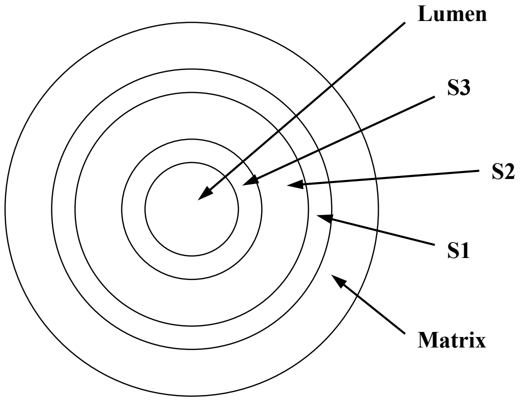
“True modulus” or “in situ” modulus?

Fiber or aligned fiber assembly: CCA models

Fiber with homogenized fiber wall



Fiber with all fiber wall layers



Volume fraction of the lumen in fiber

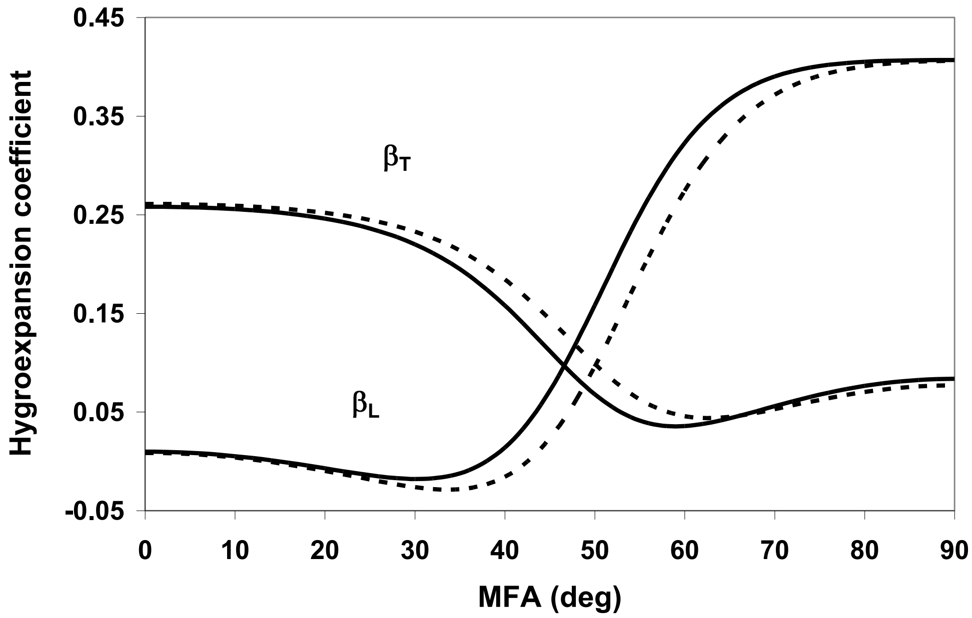
$$V_{lumen} = 0.36$$

Resin Properties

$$E_m = 3.0GPa$$

$$\nu_m = 0.35$$

Example : hygroexpansion of a fiber

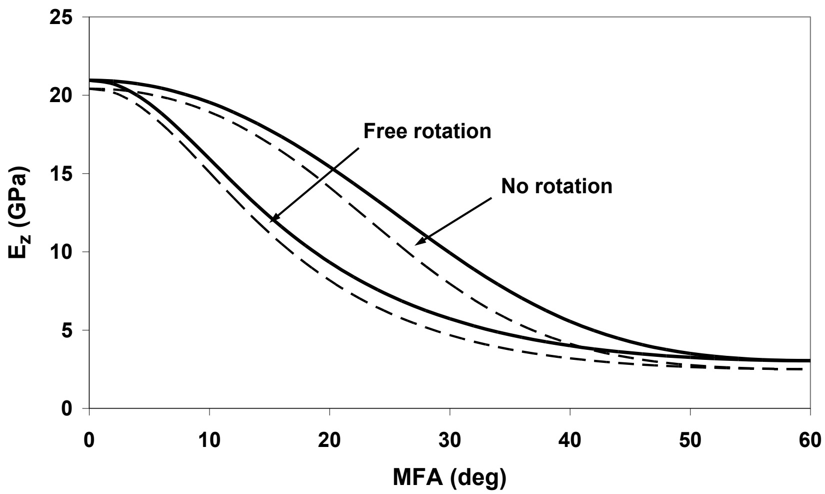


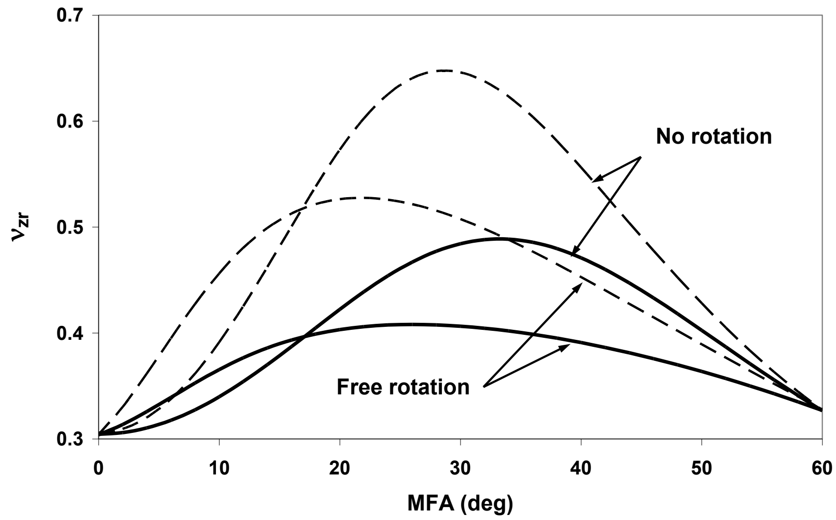
Solid line = individually included layers
Dashed line = homogenized layers (CLT) + CC

Fiber with Helical Structure (composites)

Two extreme cases: (i) No rotation and (ii) free rotation

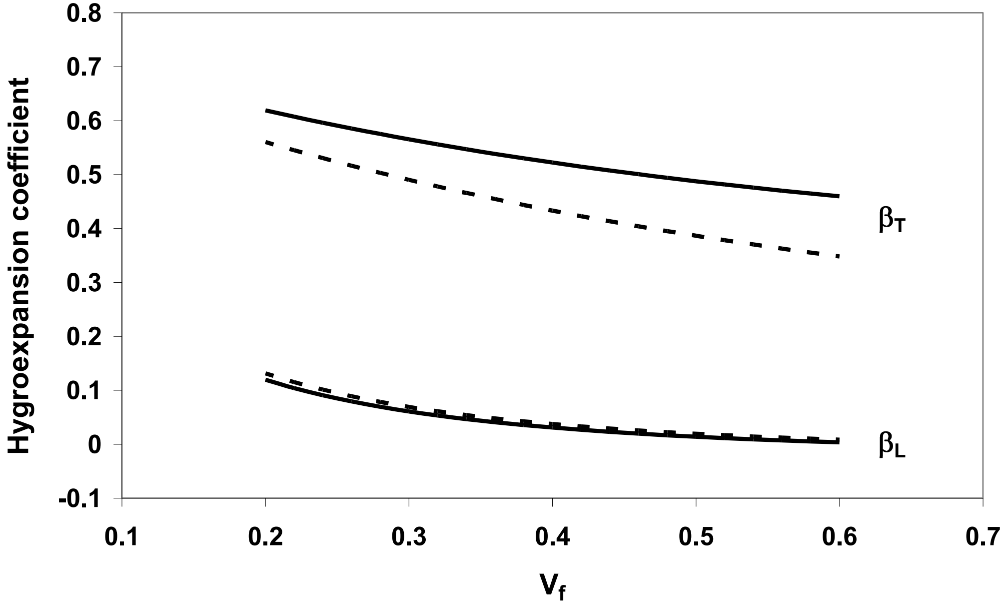
Layers	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}
S2 and S3	69.35	2.883	0.988	1.008	0.218	0.430
S1	29.58	2.948	1.012	1.015	0.288	0.452





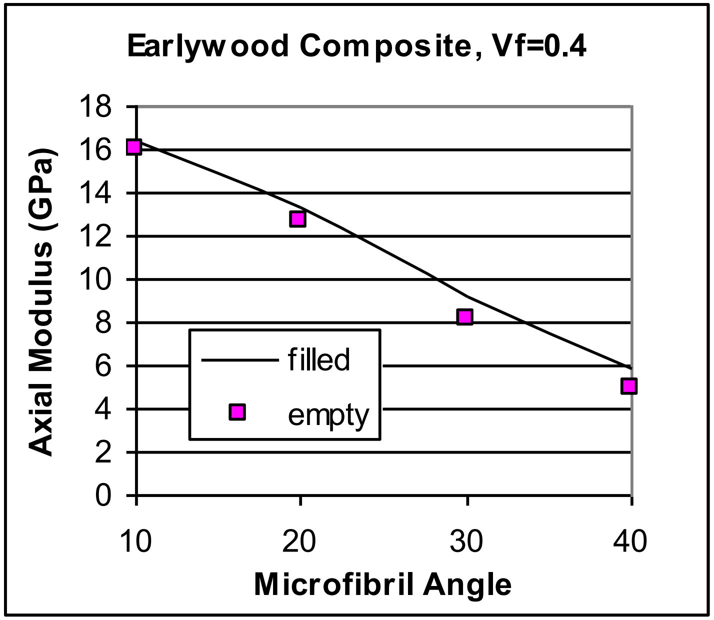
V_f = 50 % ; dashed line = empty lumen ; solid line = filled lumen

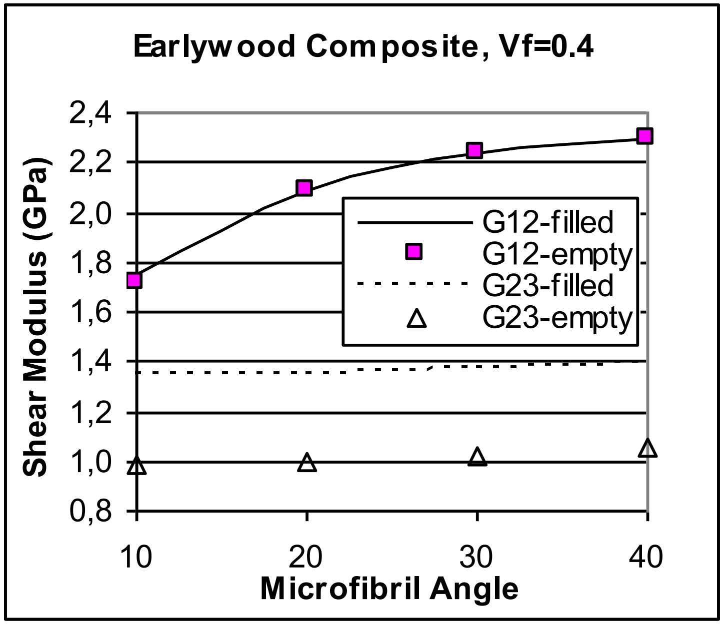
Aligned composite: hygroexpansion



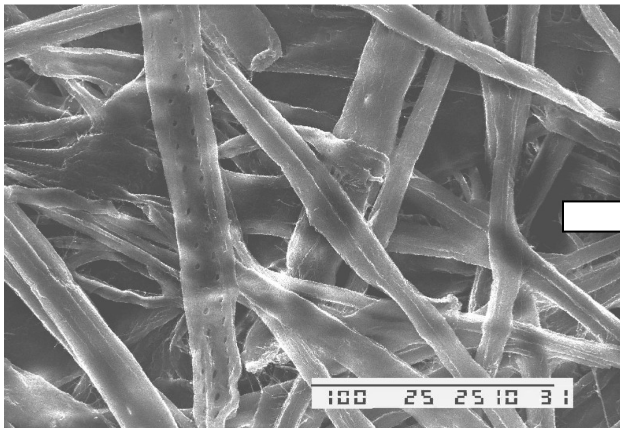
Solid line = filled lumen
Dashed line = empty lumen

Aligned composite: effect of microfibril angle in S2

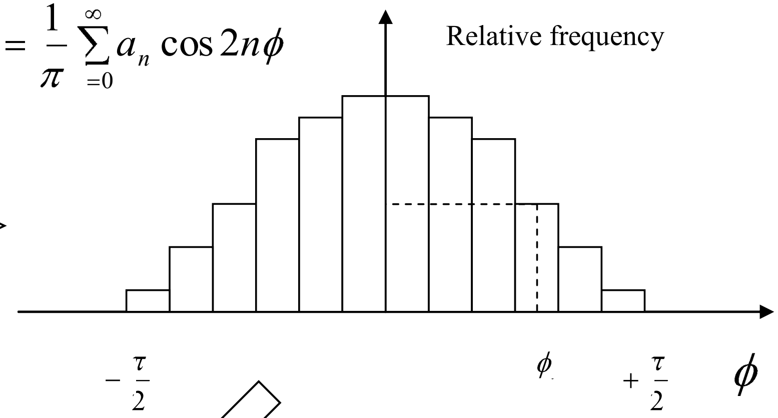


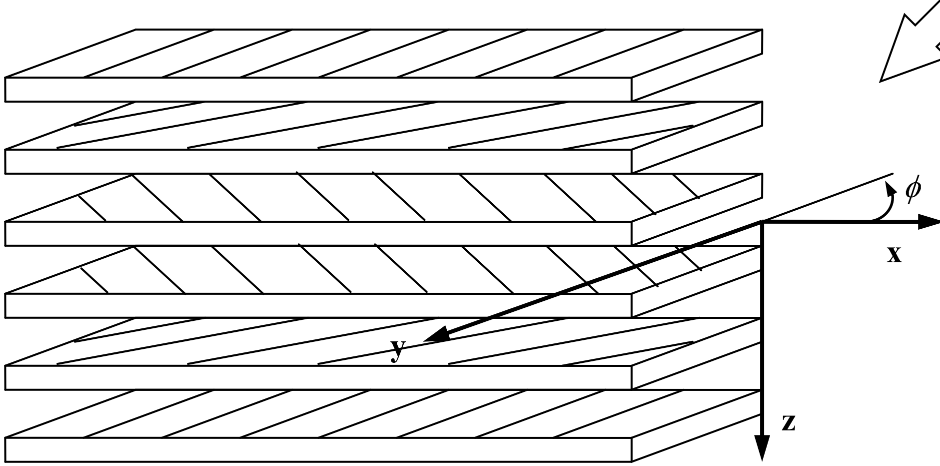


Real Composite



$$f(\phi) = \frac{1}{\pi}\sum_{n=0}^{\infty} a_n \cos 2n\phi$$





$$Q_{ij}^{comp} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \bar{Q}_{ij}(\phi) f(\phi) d\phi$$

Use the UD layer properties !

Random composites

$$Q_{11}^{comp} = Q_{22}^{comp} = \frac{1}{8}\left(3Q_{11}^{a} + 3Q_{22}^{a} + 2Q_{12}^{a} + 4Q_{66}^{a}\right)$$

$$Q_{12}^{comp} = \frac{1}{8}\left(Q_{11}^{a} + Q_{22}^{a} + 6Q_{12}^{a} - 4Q_{66}^{a}\right)$$

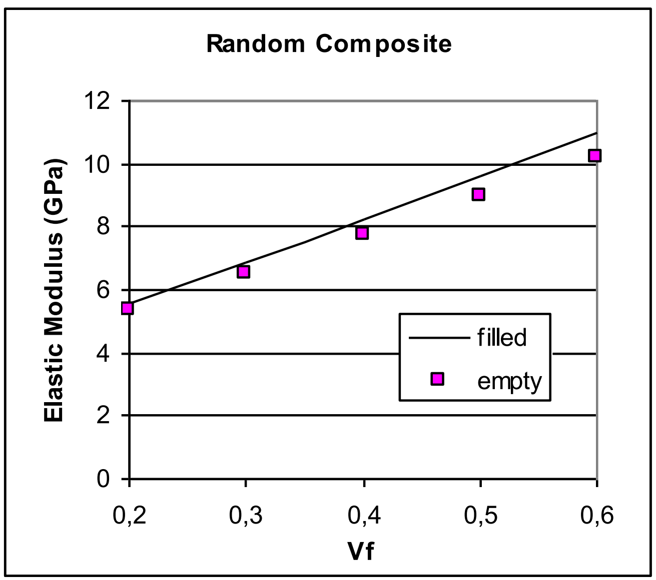
$$G^{comp} = Q_{66}^{comp}$$

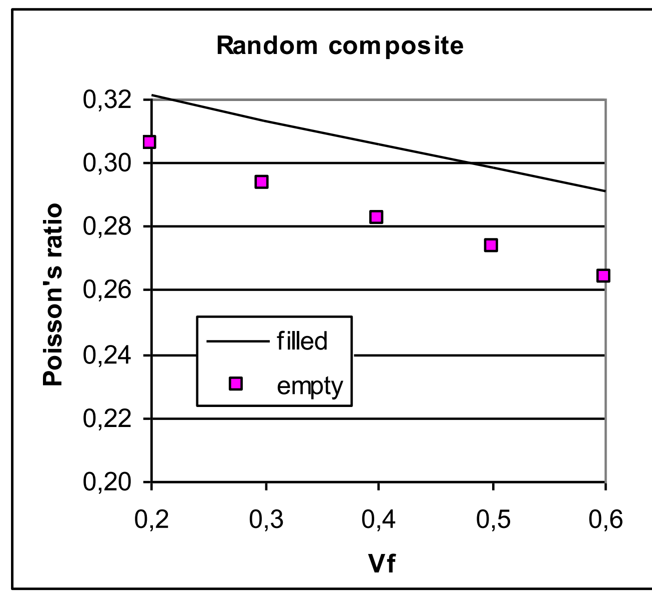
$$\nu^{comp} = \frac{Q_{12}^{comp}}{Q_{11}^{comp}}$$

$$E^{comp} = 2G^{comp}\left(1 + \nu^{comp}\right)$$

Effect of fiber content

$$V_{lumen} = 0.36$$





Conclusions

- The hierarchical structure of the natural fiber composite with large differences in scales justifies the use of the multiscale approach
- Models covering the scales from chemical constituents to macrocomposite with a given fiber orientation distribution are applied
- Simple “rule of mixtures” models may be successfully used in many cases