Tensile strength of single fibers: test methods and data analysis

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Outline

• motivation
• fiber strength as a statistical variable
• inconsistency in Weibull distribution parameters
• modified Weibull distribution
• example 1: glass fibers
• example 2: flax fibers
• tensile strength of fiber-reinforced composite
Fiber tensile strength
determines the strength of a composite material
strength at ineffective length $L$
Weibull distribution (1939)

For Poisson distribution of flaws in a fiber with power-law intensity 

\[ n = \frac{1}{l_0} \left( \frac{\sigma}{\beta} \right)^\alpha \]

average number of flaws with strength ≤ \( \sigma \)

\[ \langle N(l, \sigma) \rangle = \frac{l}{l_0} \left( \frac{\sigma}{\beta} \right)^\alpha \]

and fiber strength distribution

\[ P(\sigma) = 1 - \exp \left[ -\frac{l}{l_0} \left( \frac{\sigma}{\beta} \right)^\alpha \right] \]
Fiber tension test (FTT)

Output: fiber strength

+ straightforward method for long fibers

- labor intensive
- stress concentration at fiber ends
- complicated for short gauge length
Inconsistency of Weibull distribution parameters

\[ P(\sigma) = 1 - \exp \left[ -\frac{l}{l_0} \left( \frac{\sigma}{\beta} \right)^\alpha \right] \quad \Rightarrow \quad (\alpha_1, \beta_1) \]

\[ \langle \sigma \rangle = \beta (l/l_0)^{-1/\alpha} \Gamma(1 + 1/\alpha) \quad \Rightarrow \quad (\alpha_2, \beta_2) \]

\( \alpha_2 \geq \alpha_1 \)
<table>
<thead>
<tr>
<th>Fiber type</th>
<th>Shape parameter of Weibull strength distribution</th>
<th>Ratio $\alpha_1/\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>at a fixed fiber length, $\alpha_1$</td>
<td>from strength-length data, $\alpha_2$</td>
</tr>
<tr>
<td>Carbon Torey T800</td>
<td>4.5</td>
<td>6.4</td>
</tr>
<tr>
<td>(Okabe et al., 2002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E-glass Nito Boseki</td>
<td>5.4</td>
<td>9</td>
</tr>
<tr>
<td>(Anderssons et al., 2002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flax FinFlax (Anderssons et al., 2005)</td>
<td>2.8</td>
<td>5.2</td>
</tr>
<tr>
<td>Flax EkoFlax (Anderssons et al., 2009)</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>
Does the discrepancy of \( \alpha_1, \alpha_2 \) matter?

**E-glass fibers**

**Mean strength as a function of length**

\[
\langle \sigma \rangle = \beta \left( \frac{l}{l_0} \right)^{-1/\alpha} \Gamma \left( 1 + \frac{1}{\alpha} \right)
\]

**Strength distribution at a fixed length**

\[
P(\sigma) = 1 - \exp \left[ -\frac{l}{l_0} \left( \frac{\sigma}{\beta} \right)^\alpha \right]
\]

Discrepancy in strength interpolation

\( \alpha_1 = 5.4 \)

\( \alpha_2 = 9 \)

Discrepancy in fracture probability

\( \alpha_1 = 5.4 \)

\( \alpha_2 = 9 \)
The modified Weibull distribution

\[
P(\sigma) = 1 - \exp \left[ - \left( \frac{l}{l_0} \right)^\gamma \left( \frac{\sigma}{\beta} \right)^\alpha \right]
\]

\[
\langle \sigma \rangle = \beta \left( \frac{l}{l_0} \right)^{-\gamma/\alpha} \Gamma(1 + 1/\alpha)
\]

*Why classical Weibull distribution may not apply:*

- Watson and Smith (1985): diameter variation between fibers
- Beyerlein and Phoenix (1996): variations in material texture from fiber to fiber
- Jeulin (1996): large-scale fluctuation of the density of defects in fibres
- Curtin (2000): each fiber has a Weibull strength distribution with a random scale parameter
The modified Weibull distribution: interpretation

\[
\begin{align*}
 n_i(\sigma) &= \frac{1}{l_0} \left( \frac{\sigma}{b_i} \right)^a \\
n_2(\sigma) &= \frac{1}{l_0} \left( \frac{\sigma}{b_2} \right)^a \\
\cdots \\
n_k(\sigma) &= \frac{1}{l_0} \left( \frac{\sigma}{b_k} \right)^a
\end{align*}
\]

Defect density

\[
P_i(\sigma) = 1 - \exp \left[ - \frac{l}{l_0} \left( \frac{\sigma}{b_i} \right)^a \right]
\]

Strength distribution of fiber batch

Probability of picking an \(i\)th-type fiber

Probabilty of picking an \(i\)th-type fiber

Strength distribution of a given fiber
Strength distribution of an individual fiber

\[ P(\sigma) = 1 - \exp\left[ -\frac{l}{l_0} \left( \frac{\sigma}{b} \right)^a \right] \]

Scale parameter distribution among fibers in a batch

\[ P(b) = 1 - \exp\left[ -\left( \frac{b}{B} \right)^m \right] \]

Extensive numerical simulations with various \( a, m, B \) values

Curtin (2000) *J Compos Mater*

Strength distribution of fiber batch

\[ P(\sigma) = 1 - \exp\left[ -\left( \frac{l}{l_0} \right)^\gamma \left( \frac{\sigma}{\beta} \right)^\alpha \right] \]

\[ \gamma = \frac{m}{\sqrt{m^2 + a^2}} \quad \alpha = \frac{ma}{\sqrt{m^2 + a^2}} \]

\[ \beta = \left( 1 - (m^2 + a^2)^{-0.75} \right) B \]
Fiber fragmentation test (FFT)

Output: number of fiber breaks as a function of applied strain

+ yields strength distribution from a single test
+ characterizes an individual fiber

- residual strain estimate needed
- for non-linear elastic fibers, fiber stress-strain response needed to convert limit strain to stress
SFF specimen preparation

- Aluminium Frame
- Tape
- Fibers
- Resin
- Mould Covered by Teflon
- Silicon Tube Sealing

Specimen:
- Clamping area
- Fiber
- Matrix block

Dimensions:
- 25-35 mm
- 7-9 mm
- 2 mm
- 3-4 mm
SFF testing

MINIMAT (Miniature Testing Machine)

Motor
Load cell
Grips
Strain Gage
Specimen
Microscope and Video Camera
MINIMAT (Miniature Testing Machine)
Electronic Unit
Amplifier
Computer IBM PC
Video Recorder
Video Monitor
Glass fiber fragmentation data

\[ P(\sigma) = 1 - \exp \left[ -\frac{l}{l_0} \left( \frac{\sigma}{b} \right)^a \right] \]

Linear elastic fibers, hence \( \sigma = E\varepsilon \)

Number of breaks vs strain

\[ n = \frac{1}{l_0} \left( \frac{E\varepsilon}{b} \right)^a \]

\( a = 8.2 \)

\( b = 3200 \text{ MPa} \)
Glass fiber fragmentation data

Number of breaks vs strain for several fibers

\[ P(\sigma) = 1 - \exp \left[ - \frac{l}{l_0} \left( \frac{\sigma}{b} \right)^a \right] \]

Fragmentation diagrams: \( n_i = \frac{1}{l_0} \left( \frac{E \varepsilon}{b_i} \right)^a \)

✓ the same slope (parameter \( a \))
✓ location (parameter \( b \)) exhibits scatter
Individual glass fibers characterised, by FFT, by

\[ P(\sigma) = 1 - \exp \left[ -\frac{l}{l_0} \left( \frac{\sigma}{b} \right)^a \right] \]

\[ a = 8.4 \]
\[ B = 3150 \text{ MPa} \]
\[ m = 10.6 \]

Fiber batch strength distribution

\[ P(\sigma) = 1 - \exp \left[ -\left( \frac{l}{l_0} \right)^\gamma \left( \frac{\sigma}{\beta} \right)^\alpha \right] \]

Curtin’s relations (2000)

\[ \gamma = \frac{m}{\sqrt{m^2 + a^2}} \approx 0.78 \]
\[ \alpha = \frac{m a}{\sqrt{m^2 + a^2}} \approx 6.7 \]
\[ \beta = \left(1 - \left(m^2 + a^2\right)^{-0.75}\right)B \approx 3080 \text{ MPa} \]
Glass fiber strength: SFT and SFF

Distribution parameters determined by SFF

\[ P(\sigma) = 1 - \exp \left[ -\left( \frac{l}{l_0} \right)^\gamma \left( \frac{\sigma}{\beta} \right)^\alpha \right] \]

\[ \langle \sigma \rangle = \beta (l/l_0)^{-\gamma/\alpha} \Gamma(1 + 1/\alpha) \]
Elementary flax fiber

length ~ 3 cm
diameter ~ 20 µm
Flax fiber SFF

\[ P(\varepsilon) = 1 - \exp \left( -\frac{l}{l_0} \left( \frac{\varepsilon}{b} \right)^{a_\varepsilon} \right) \]

Mechanical loading, \( \varepsilon = 1\% \)

- \( E_{ext} = E_m \), \( v_{ext} = v_m \)
- \( E_{ext} = E/2 \), \( v_{ext} = v_f \)
- \( E_{ext} = E_f \), \( v_{ext} = v_f \)

\[ P(\varepsilon) = 1 - \exp \left( -\frac{l}{l_0} \left( \frac{\varepsilon}{b} \right)^{a_\varepsilon} \right) \]

\[ n = \frac{1}{l_0} \left( \frac{\varepsilon}{b} \right)^{a_\varepsilon} \]
Elementary flax fiber: response to axial tension

\[ \sigma = E (\varepsilon - \varepsilon_n) \]

\[ \langle \sigma \rangle = \langle E \rangle \left( \langle \varepsilon \rangle - \langle \varepsilon_n \rangle \right) \]

\[ E = 69 \pm 20 \text{ GPa} \]

\[ \varepsilon_n = 0.32 \pm 0.42\% \]
Flax fiber average strength: SFT and SFF

\[ \langle \sigma \rangle = \langle E \rangle (\langle \varepsilon \rangle - \langle \varepsilon_n \rangle) \]

\[ \langle \varepsilon \rangle = \beta_{\varepsilon} \left( \frac{l}{l_0} \right)^{\frac{\gamma_{\varepsilon}}{\alpha_{\varepsilon}}} \Gamma \left( 1 + \frac{1}{\alpha_{\varepsilon}} \right) \]

The modified Weibull failure strain distribution parameters from SFF:

- \( \gamma_{\varepsilon} = 0.79 \)
- \( \alpha_{\varepsilon} = 4.97 \)
- \( \beta_{\varepsilon} = 2.5\% \)
\[ \varepsilon_{\text{FTT}} = \varepsilon_{\text{FFT}} \]

Man-made fibers

Natural organic fibers

\[ \varepsilon_{\text{FTT}} = \varepsilon_{\text{FFT}} \]
The effect of kink bands on bast fiber strength

Marginal effect:

- no correlation between the amount of kink bands and fiber strength
  Baley (2004); flax
  Thygesen et al. (2007); hemp
  Andersons et al. (2009); flax

Determining effect:

- absence of kink bands increases mean strength of flax by ~20%
  Bos (2002)

- failure in tension initiates within a kink band in flax
  Khalili et al. (2002)
  Lamy, Pomel (2002)
  Baley (2004)
Derivation of bast fiber strength distribution

Probability of fracture initiated by defects: Todinov (2000)

\[ P_d(\sigma) = 1 - \exp\left[-\left(\frac{\sigma}{b}\right)^a\right] \]

number of defects; in our case \( \lambda = \frac{l}{s} \)

defect failure probability; approximately

\[ P(\sigma) = 1 - \exp[-\lambda P_d(\sigma)] \]

Strength distribution

\[ P(\sigma) = 1 - \exp\left[-\frac{l}{s}\left(\frac{\sigma}{b}\right)^a\right] \]

Mean strength

\[ \langle \sigma \rangle = b\left(\frac{s}{l}\right)^{\frac{1}{a}} \Gamma\left(1+\frac{1}{a}\right) \]
Derivation of fiber batch strength distribution: random $s$

Strength distribution of an individual fiber

$$P(\sigma) = 1 - \exp \left[- \frac{l}{s} \left(\frac{\sigma}{b}\right)^a\right]$$

Kink spacing distribution among fibers in a batch

$$P(s) = 1 - \exp \left[- \left(\frac{s}{\bar{s}}\right)^{m}\right]$$

Curtin (2000)

WoW model

Strength distribution of fiber batch

$$P(\sigma) = 1 - \exp \left[- \left(\frac{l}{l_0}\right)^{\gamma} \left(\frac{\sigma}{\beta}\right)^a\right]$$

$$\gamma = \frac{m}{\sqrt{m^2 + 1}} \quad \alpha = \frac{ma}{\sqrt{m^2 + 1}}$$

$$\beta = \left(1 - a^{-1.5} \left(p^2 + 1\right)^{-0.75}\right)b\left(\frac{\bar{s}}{l_0}\right)^{\frac{1}{\alpha}}$$
Elementary flax fibers, Finflax

Kink spacing distribution (5 mm fibers)

\[ P(s) = 1 - \exp \left( - \left( \frac{s}{\bar{s}} \right)^m \right) \]

\( m = 1.4 \)
\( \bar{s} = 0.21 \)

\[ \gamma = \frac{m}{\sqrt{m^2 + 1}} \approx 0.8 \]

Fiber strength distribution

\[ P(\sigma) = 1 - \exp \left( - \left( \frac{l}{l_0} \right)^\gamma \left( \frac{\sigma}{\beta} \right)^\alpha \right) \]

\( \alpha = 2.9 \)
\( \beta = 1700 \text{ MPa} \)
Elementary flax fibers, Ekotex

Kink spacing distribution

\[ P(s) = 1 - \exp \left( -\left( \frac{s}{\bar{s}} \right)^m \right) \]

- \( m = 5.2 \)
- \( \bar{s} = 0.07 \)

\[ \gamma = \frac{m}{\sqrt{m^2 + 1}} \approx 0.98 \]

Fiber strength distribution

\[ P(\sigma) = 1 - \exp \left[ -\left( \frac{L}{L_0} \right)^{\frac{1}{\beta}} \left( \frac{\sigma}{\beta} \right)^{\alpha} \right] \]

- \( \alpha = 3.1 \)
- \( \beta = 1400 \text{ MPa} \)
Flax-fiber reinforced composites

RTM flax mat / thermoset polymer

Extruded flax / PP and MAPP

Fiber length ~ few cm
Fiber orientation ~ in-plane random

Fiber length ~ 1 mm
Fiber orientation ~ 3D random
Composite stiffness: model

Cox - Krenchel model:

\[ E = \eta_{oE} \eta_{lE} E_f \nu_f + E_m \nu_m \]

\[
\eta_{lE} = \frac{1}{\langle l \rangle} \int_{l_{\text{min}}}^{l_{\text{max}}} l \cdot \left[ 1 - \frac{\tanh \xi(l/2)}{\xi(l/2)} \right] \cdot h(l) dl
\]

\[
\xi = \sqrt{\frac{2G_m}{r_o^2 E_f \ln(R/r_0)}}
\]

extruded composite (3D distribution): \( \eta_{oE} = 1/5 \)

FFM composite (2D distribution): \( \eta_{oE} = 3/8 \)
Composite stiffness: results

Extruded flax / thermoplastic matrix composites

Flax fiber mat / thermoset matrix composites

$E$, MPa

Predicted

$\nu_f = 0.3$
Composite strength: model

Modified Fukuda-Chou critical zone strength theory

\[ \sigma_c = \eta_s \sigma_{uf} \nu_f + \left(1 - \nu_f\right)\sigma_m \]

- all fibers bridging the critical zone and the matrix fail simultaneously
- fiber stress at failure proportional strength, reduced by ineffective length

\[ \sigma_0(l) = \begin{cases} \left(1 - \frac{l_c}{2l}\right)\sigma_{uf} & l \geq l_c \\ \frac{l}{2l_c}\sigma_{uf} & l < l_c \end{cases} \]

\[ \sigma_{uc} = \nu_f \int_{l_n}^{l_0} \int_\theta \frac{\sigma_0(l)l \cos^4 \theta}{\langle l \rangle} h(l) g(\theta) d\theta dl + \left(1 - \nu_f\right)\sigma_m \]
Composite strength: theoretical vs experimental

Flax fiber mat / thermoset matrix composites

Extruded flax / thermoplastic matrix composites
The critical zone width is chosen equal to the critical fibre length \( l_n = l_c \)

\[
\sigma_c = \eta_s \frac{V_f}{5} \frac{\sigma_{uf}(l)}{\langle l \rangle} \left[ 1 - \left( \frac{l_c}{l} \right)^5 \right] \left( 1 - \frac{l_c}{2l} \right) h(l) dl + (1 - \nu_f) \sigma_m
\]

\[
\sigma_{uf}(l) = \beta \sigma (l/l_0)^{-\gamma/\alpha} \Gamma(1 + 1/\alpha)
\]

\[
l_c = \left( \frac{\beta \sigma \Gamma(1 + 1/\alpha)r_f}{\nu l_0^{-\gamma/\alpha}} \right)^{\alpha/(\gamma + \alpha)}
\]

\[
\sigma_m = \frac{\sigma_{uf} E_m}{E_f}
\]
Composite strength: Flax fiber mat / thermoset

\[
\sigma_c = \eta_i \frac{V_f}{4\pi} \int \frac{\sigma_{uf}(l)}{\langle l \rangle} \left( 3\cos^{-1} \frac{l_c}{l} + \frac{l_c}{l} \left( 3 + 2 \left( \frac{l_c}{l} \right)^2 \right) \sqrt{1 - \left( \frac{l_c}{l} \right)^2} \left( 1 - \frac{l_c}{2l} \right) b(l) dl + (1 - \nu_f) \sigma_m \right)
\]

\( \nu_f = 0.3 \)

\[
\sigma_{uf}(l) = \beta_\sigma \left( \frac{l}{l_0} \right)^{-\gamma/\alpha} \Gamma(1 + 1/\alpha)
\]

\[
l_c = \left( \frac{\beta_\sigma \Gamma(1 + 1/\alpha) r_f}{\tau l_0^{-\gamma/\alpha}} \right)^{\alpha/(\gamma + \alpha)}
\]

\[
\sigma_m = \frac{\sigma_{uf} E_m}{E_f}
\]
Conclusions

- The modified Weibull distribution for fiber strength should be used if the variation of strength with length is of interest.

- To evaluate distribution parameters by fiber tension tests alone, strength data for at least two gauge lengths are needed.

- Alternatively, fiber fragmentation tests can be applied to obtain strength distribution of brittle man-made fibers (glass) and average strength-length relation for bast fibers (flax).

- The modified Weibull distribution enables accurate scaling of fiber strength from tests at length of cm range to mm range, needed for composite strength analysis.
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