How the relationship between wood density and shrinkage depends on the microstructure of the cell wall

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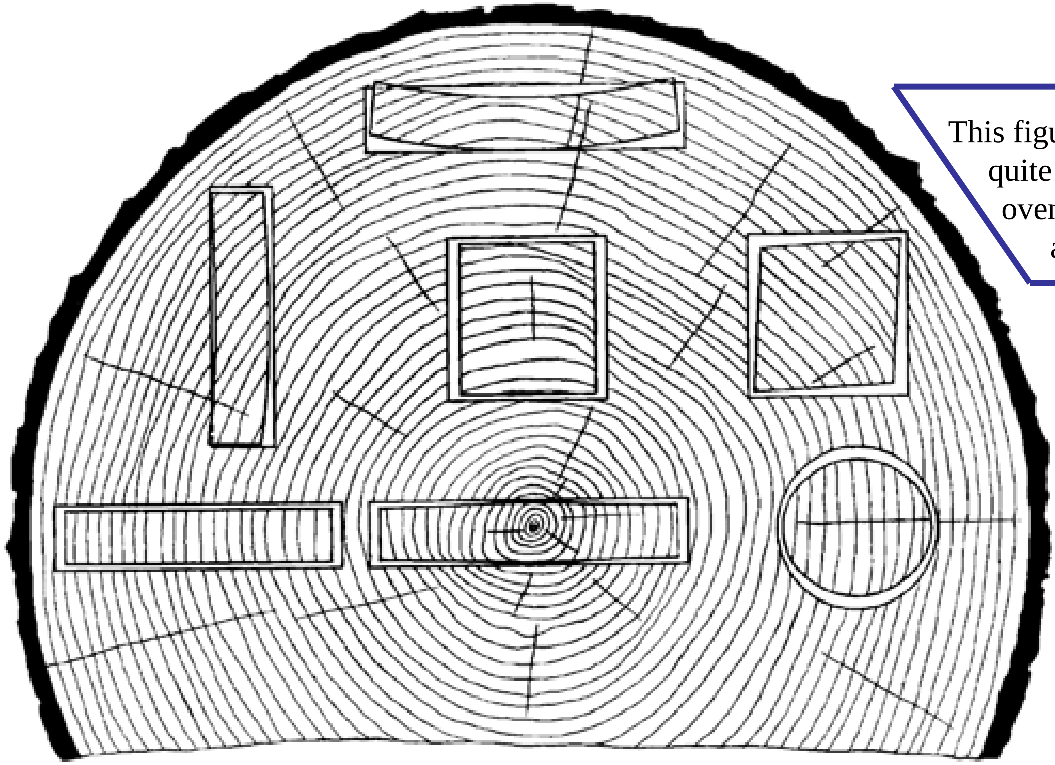
Every discussion of wood shrinkage brings up two issues early on:
1) Shrinkage is anisotropic, tangential greater than radial.
2) Shrinkage (volumetric) is roughly proportional to density.



This figure is quantitatively quite realistic for green to oven dry shrinkage of an average conifer wood

This figure appeared in the first edition of the Wood Handbook (1935) and has appeared in all editions since and in many many other publications.

Caption in first edition: Characteristic shrinkage and distortion of flats, squares, and rounds ("flat, square and round pieces" since 1999) as affected by the direction of the annual rings. Tangential shrinkage is about twice as great as radial.

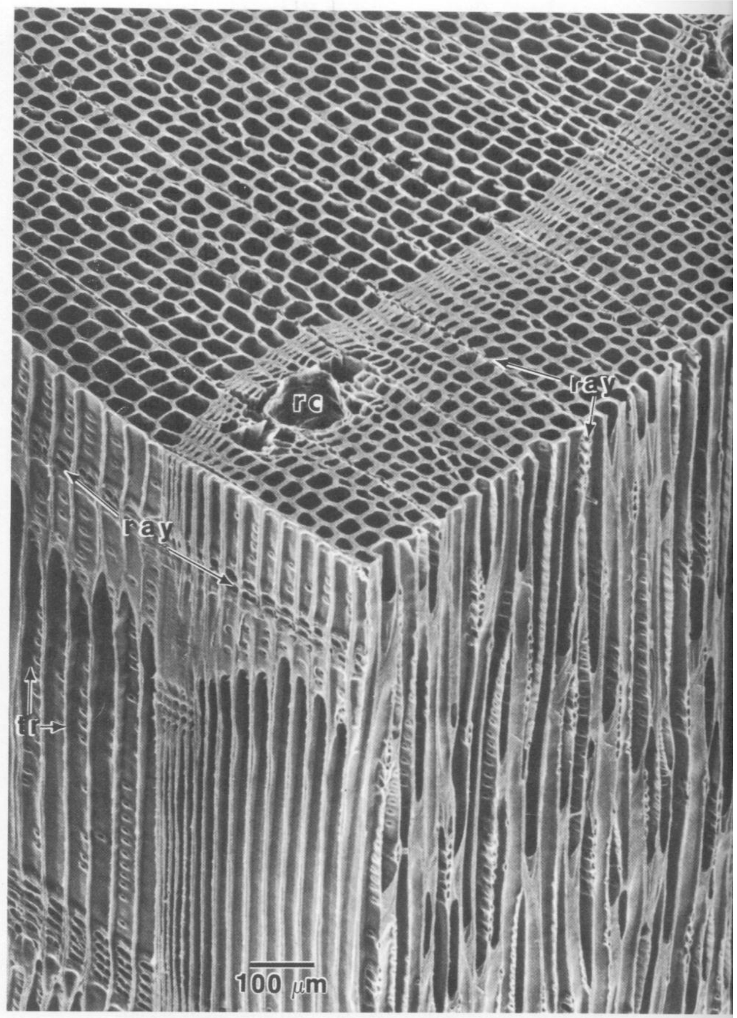


Figure 1. Scanning electron micrograph of a cube of eastern white pine microtomed on three surfaces. Note the arrangement of longitudinal tracheids (tr) in radial files and the structure of the rays. In this species resin canals (rc) are rather prominent features.

What about the second question?

Every discussion of wood shrinkage brings up two issues early on:

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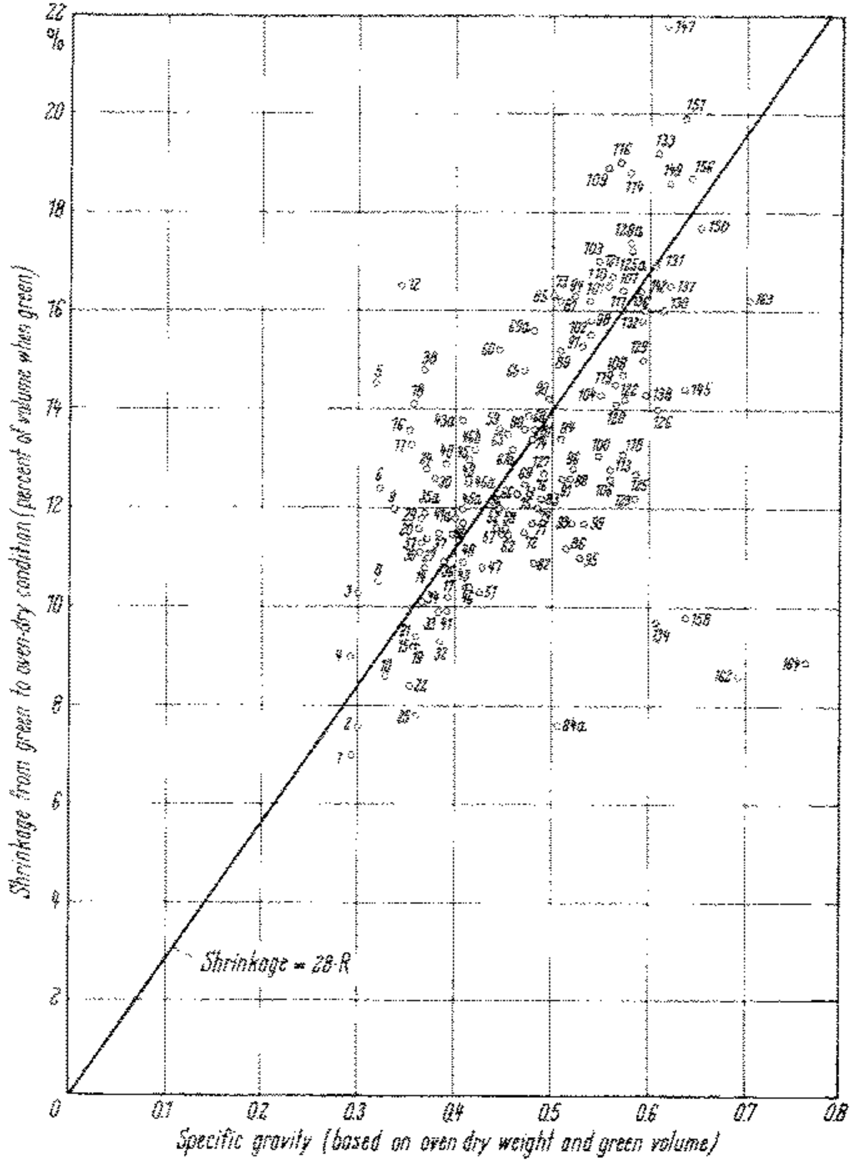


J.A. Newlin and T.R.C. Wilson, The relation of the shrinkage and strength properties of wood to its specific gravity. USDA Bulletin No.676 (1919).

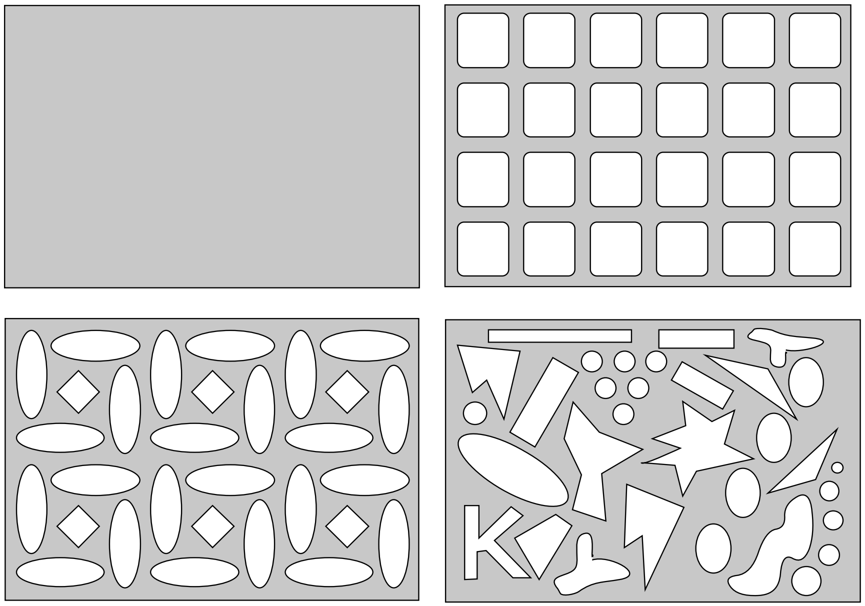
No big discussion in text books, just a simple statement like:

"High density woods have proportionately more cell wall and less lumen volume, and so shrink and swell more."

J.C.F. Walker, Primary Wood Processing – Principles and Practice, 2nd ed, Springer, 2006, p.98

End of story

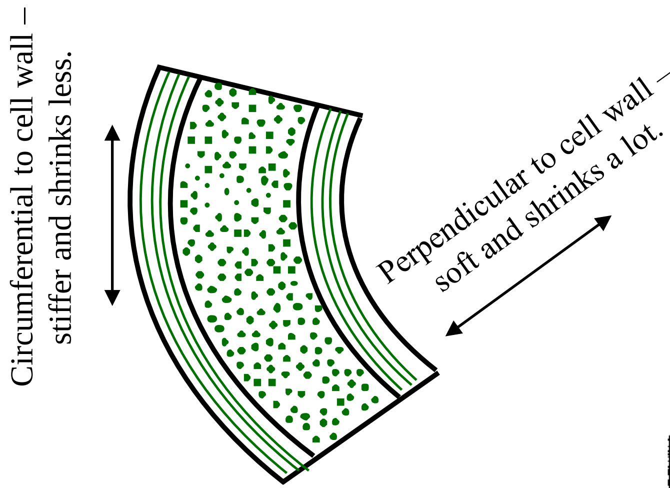
That just doesn't wash!

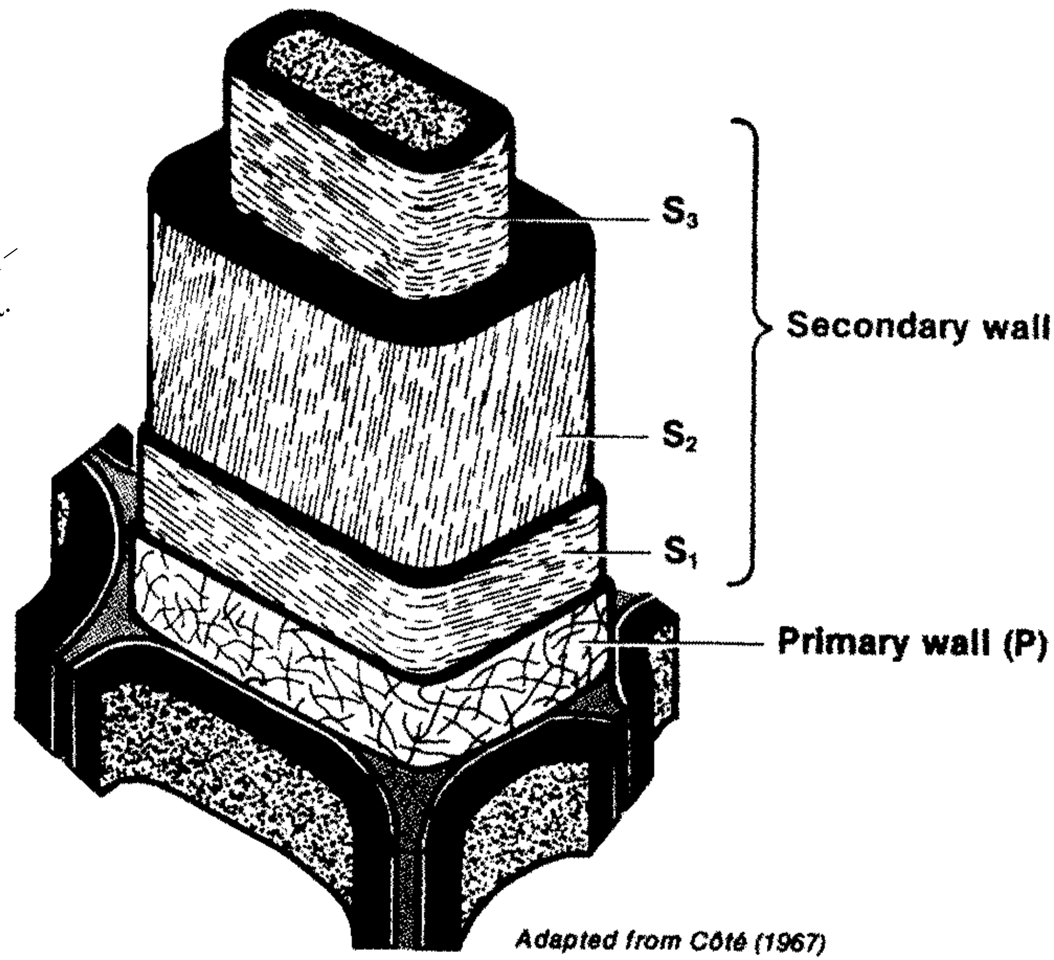


If the matrix surrounding the pores had no microstructure (or micro-structure small compared to pore dimensions) then each of the four "material" bodies here would shrink (swell) to the same outer dimensions.

The fact that denser woods tend to shrink more than lighter woods <u>must</u> be related to the microstructure of the wood at the geometric scale of the lumina. This was clearly understood 50 years ago by Stamm and Loughborough* and nicely explained by them qualitatively – but they have apparently been ignored. So I'll give it a try.

- *A.J. Stamm and W.K. Loughborough, Variation in Shrinking and Swelling of Wood. *Trans ASME* (1942), 379-386.

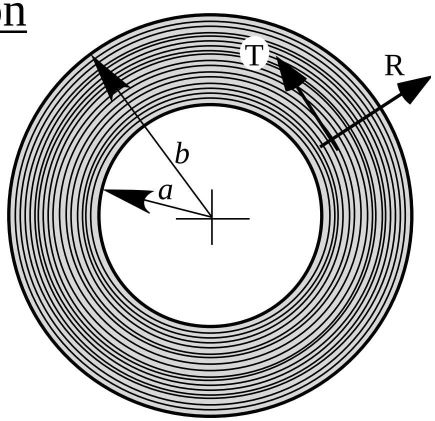


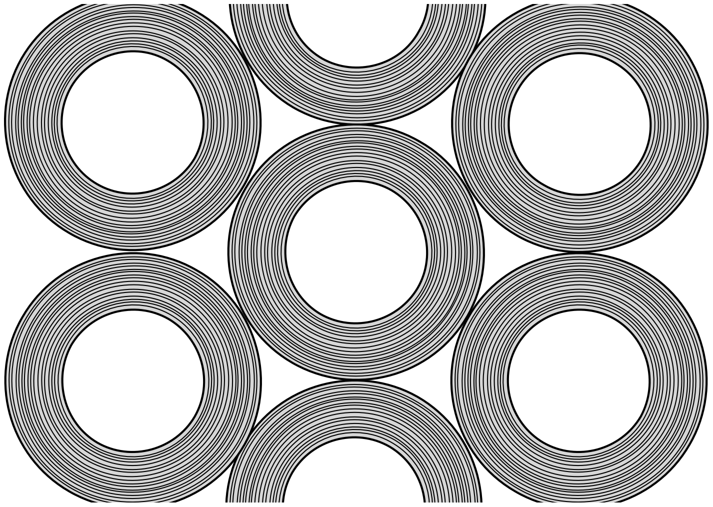


Adapted from Côté (1967)

Now let's get analytical:

Model the wood cells as cylindrical tubes*. We will homogenize the properties in the cross-section. T and R here refer to radial and tangential in the **cell** cross-section





We can exactly calculate the shrinkage of the outer diameter (= half the volumetric shrinkage) as a function of the physical parameters of the cell wall material. These are **(considering the cell wall as a material)**:

α_R – radial shrinkage, α_T – tangential shrinkage
E_R – radial elastic modulus, E_T – tangential elastic modulus, ν_{TR} and ν_{RT} – Poisson ratios
We expect $\alpha_R >> \alpha_T$ and $E_T >> E_R$

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* N. Barber, A theoretical model of shrinking wood. *Holzforschung* **22** (1968), 97-103

So we apply the equations of linear elasticity. We get a differential equation. We solve it. We apply the boundary conditions. We get

$$\frac{\Delta b}{b} = \frac{1}{\eta^2 - 1}\left[-\alpha_{\mathrm{R}}(1-\nu_{\mathrm{TR}}) + \alpha_{\mathrm{T}}(\eta^2 - \nu_{\mathrm{TR}}) + \eta^2(\alpha_{\mathrm{T}} - \alpha_{\mathrm{R}})\frac{1-\nu_{\mathrm{RT}}\nu_{\mathrm{TR}}}{c^{2\eta}-1}\left(-\frac{c^{2\eta}-c^{\eta-1}}{\eta+\nu_{\mathrm{TR}}} - \frac{1-c^{\eta-1}}{\eta-\nu_{\mathrm{TR}}}\right)\right]$$

[Δb defined as <u>reduction</u> of radius.]

$$\eta^2 = E_{\mathrm{T}}/E_{\mathrm{R}}$$

Here c is the ratio b/a and $c = \frac{1}{\sqrt{1-V}}$ where V is the volume fraction of solid material.

And note that wood specific gravity = 1.5 V. (The solid material has density ~ 1.5 g/cm^3.) Further note that the <u>volumetric</u> shrinkage will be **twice** the value given above.

This is an exact solution. Oh my goodness, so many material parameters! **What a mess**. But we only want to know about the <u>nature</u> of the shrinkage dependence on density.

So let's simplify the situation. We will take the Poisson ratio to be 0. This is a special case, but always ν is appreciably less than 1 anyway, so qualitatively the situation is not changed. We get

$$\frac{\Delta b}{b} = \frac{1}{\eta^2 - 1}\left[-\alpha_\mathrm{R} + \alpha_\mathrm{T}\eta^2 + \eta(\alpha_\mathrm{T} - \alpha_\mathrm{R})\frac{-c^{2\eta} + 2c^{\eta-1} - 1}{c^{2\eta} - 1}\right]$$

Now this is much better – but not good enough! In addition to α_R and α_T and of course c (which is directly related to the volume fraction V and hence to the wood density) we still have η (the square root of the ratio of elastic moduli) as a confusing factor.

So let's get rid of η (in terms of stiffness ratios) also.

In cellulosic materials at all geometric levels, in a plane of orthotropy, the following relationship holds quite well.

$$\frac{\alpha_1}{\alpha_2} = \frac{E_2}{E_1}$$

Here 1 and 2 are principal directions. That this holds for whole wood (1 and 2 are radial and tangential directions) was demonstrated and rationalized by Keyworth[1]; that this holds more generally (also paper and particle board) and that there is a sound physical basis for the relationship across the board was shown by Schulgasser[2,3]

1. R. Keylwerth, Formänderungen in Holzquerschnitten. *Holz als Roh- und Werkstoff* **7** (1951), 253-260.

2. K. Schulgasser, Thermal expansion of polycrystalline aggregates with texture. *Journal of the Mechanics and Physics of Solids* **35** (1987), 35-42.

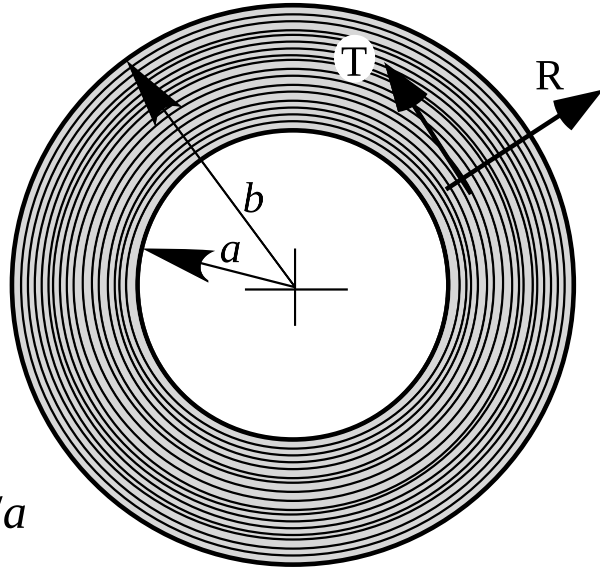
3. K. Schulgasser, Moisture and thermal expansion of wood, particle board and paper. *Paperi ja Puu* **70** (1988), 534-539.

And what we get is "simply":

$$\frac{\Delta b}{b} = \sqrt{\alpha_{\mathrm{R}} \alpha_{\mathrm{T}}} \left[\frac{c^{2\eta} - 2c^{\eta-1} + 1}{c^{2\eta} - 1} \right]$$

where now $\eta = \sqrt{\dfrac{\alpha_{\mathrm{R}}}{\alpha_{\mathrm{T}}}}$

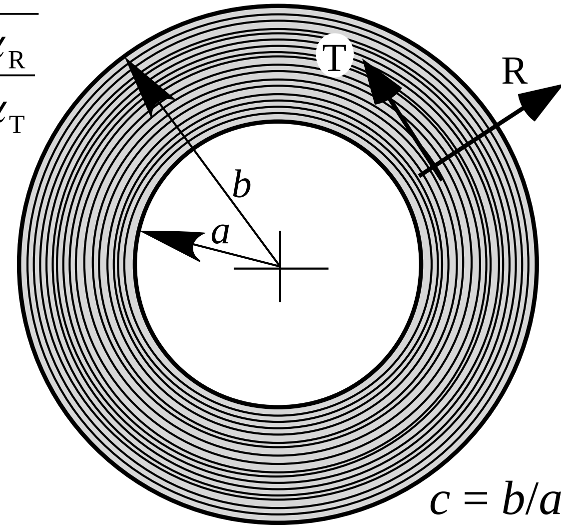
It is important to emphasize that this is an exact solution, albeit when certain (reasonable) relationships exist between physical parameters.



$c = b/a$

$$\frac{\Delta b}{b} = \sqrt{\alpha_R \alpha_T}\left[\frac{c^{2\eta} - 2c^{\eta-1} + 1}{c^{2\eta} - 1}\right]$$

$$\eta = \sqrt{\frac{\alpha_R}{\alpha_T}}$$

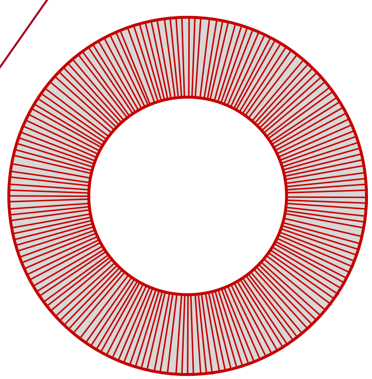


First off let's look at the two limiting cases:
$c \rightarrow 1$ (i.e. V, and thus density, very very low).
We find
$$\frac{\Delta b}{b} = \alpha_T$$

Not surprising. (Any thin-walled network of any cell shape would give this.)

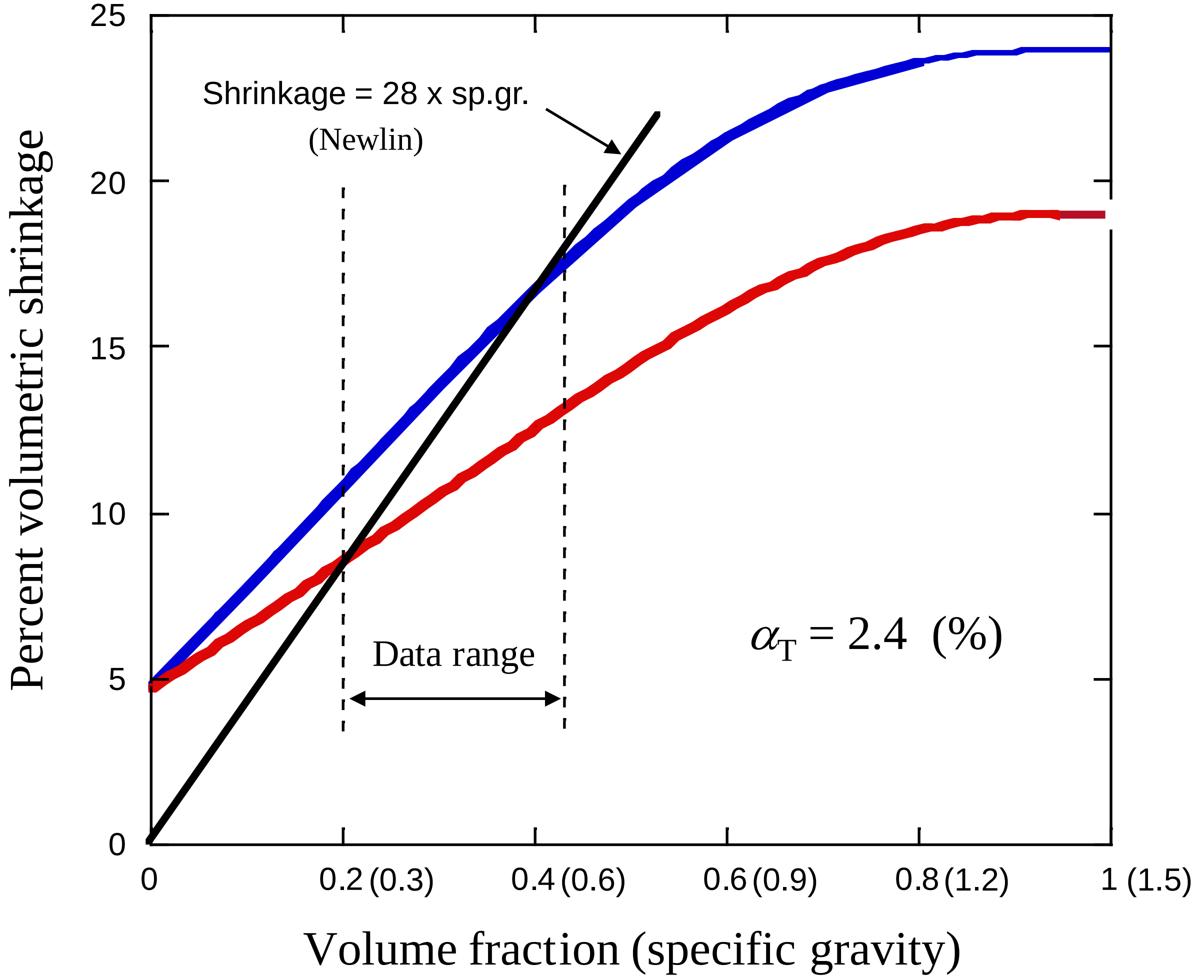
Next consider $c \rightarrow \infty$ (i.e. V approaches 1 and density 1.5).
We find
$$\frac{\Delta b}{b} = \sqrt{\alpha_R \alpha_T}$$

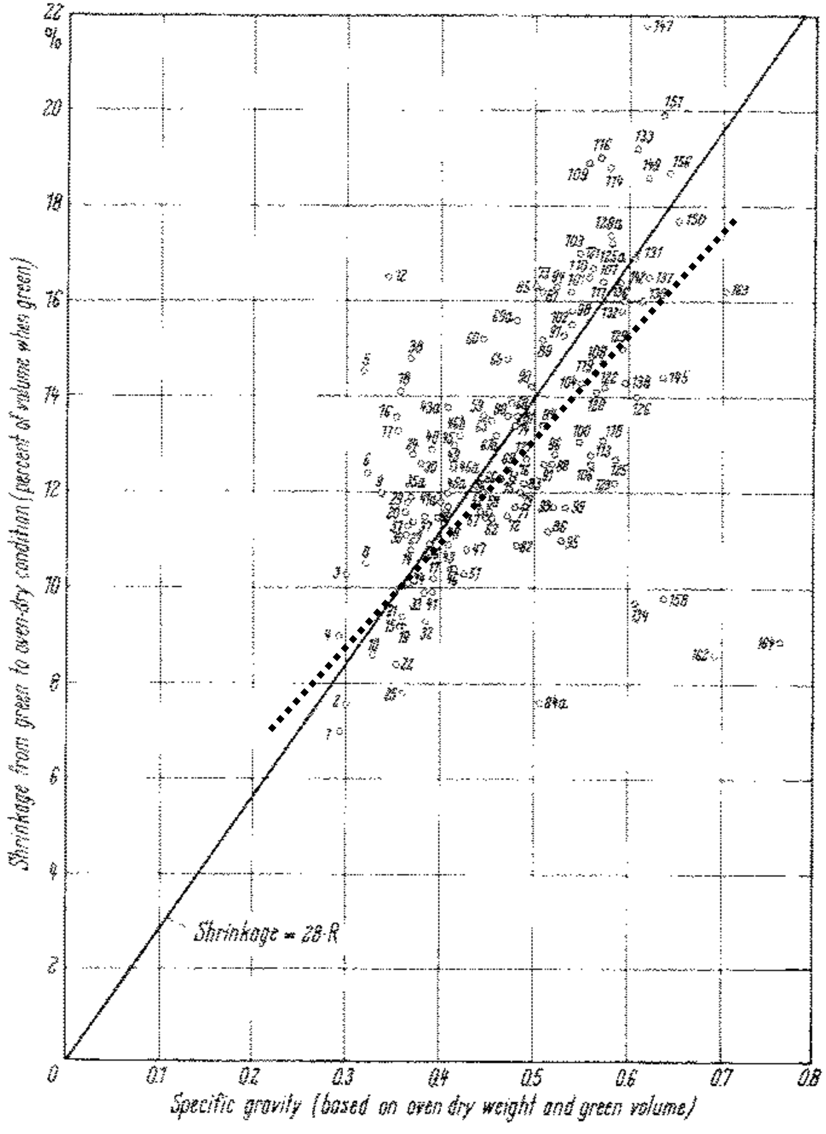
And the implication is (since $\alpha_R > \alpha_T$) that as density increases shrinkage increases.



But what if α_R had been smaller than α_T ? ? ? ?

Now let's look at how shrinkage actually depends on density.
As an example we will take $\alpha_R/\alpha_T = 25$ and $\alpha_R/\alpha_T = 16$.





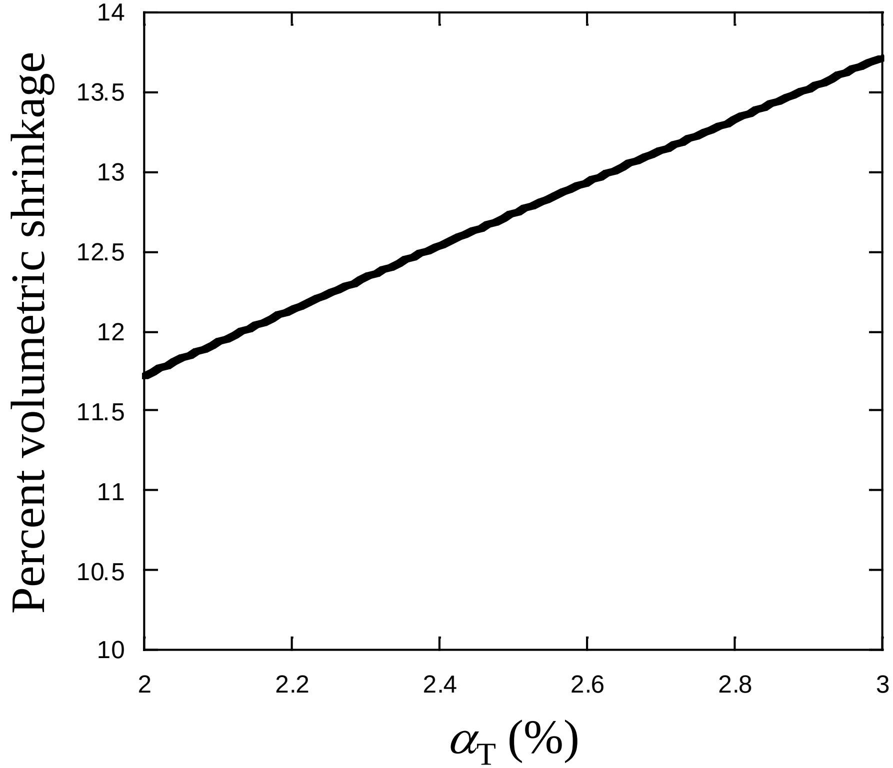
In other words the fit should indeed be more or less linear in the range of interest but should not be forced to go through zero.

J.A. Newlin and T.R.C. Wilson, The relation of the shrinkage and strength properties of wood to its specific gravity. USDA Bulletin No.676 (1919).

And we learn another interesting thing. As micro-fibril angle decreases in the S_2 layer (which generally corresponds to increasing ring number) the α_R would probably not change perceptively, but α_T will increase somewhat (S_2 microfibrils less oriented in circumferential direction), and our model predicts that this should increase shrinkage. Now with increasing ring number, generally density also increases, but the shrinkage observed seems to be somewhat greater than that attributable simply to the proportional increase of density*.

* See for instance the data in:
M. Grekin and E. Verkasalo, Variations in basic density, shrinkage and shrinkage anisotropy of Scots pine wood from mature mineral soil stands in Finland and Sweden. *Baltic Forestry* **16** (2010), 113-125.

For instance consider a typical case (volume faction 0.31 corresponding to density 0.46) for α_R = 50% we calculate the volumetric shrinkage as α_T increases (do to decreasing microfibril angle) from 2% to 3%. We see that volumetric shrinkage does increase, but not radically. And this would be additive to shrinkage increase do to greater density.

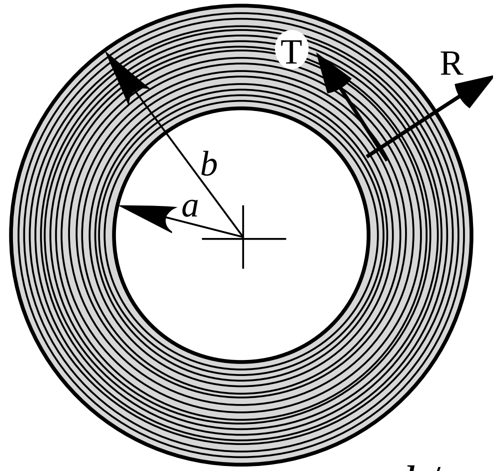


In other words a 50% increase of α_T would cause an increase of about 17% in shrinkage.

Now just out of curiosity let's ask how the lumen diameter changes on drying. (This, of course, does not influence wood global shrinkage.) We find

$$\frac{\Delta a}{a} = \sqrt{\alpha_R \alpha_T}\left[\frac{c^{2\eta} - 2c^{\eta+1} + 1}{c^{2\eta} - 1}\right]$$

[Δa defined as <u>reduction</u> of radius.]



$$\eta = \sqrt{\frac{\alpha_R}{\alpha_T}}$$

$c = b/a$

Note: There is only a *teensy weensy* difference between this expression and the one for $\Delta b/b$ on slide #15. The + in the exponent of the middle term of the numerator was there – .

Again let's look at the two limiting cases:
$c \to 1$ (i.e. V, and thus density, very very low).
We find

$$\frac{\Delta a}{a} = \alpha_T$$

Again not surprising. (Any thin-walled network of any cell shape would give this.)

Next consider $c \to \infty$ (i.e. V approaches 1 and density 1.5).
We find

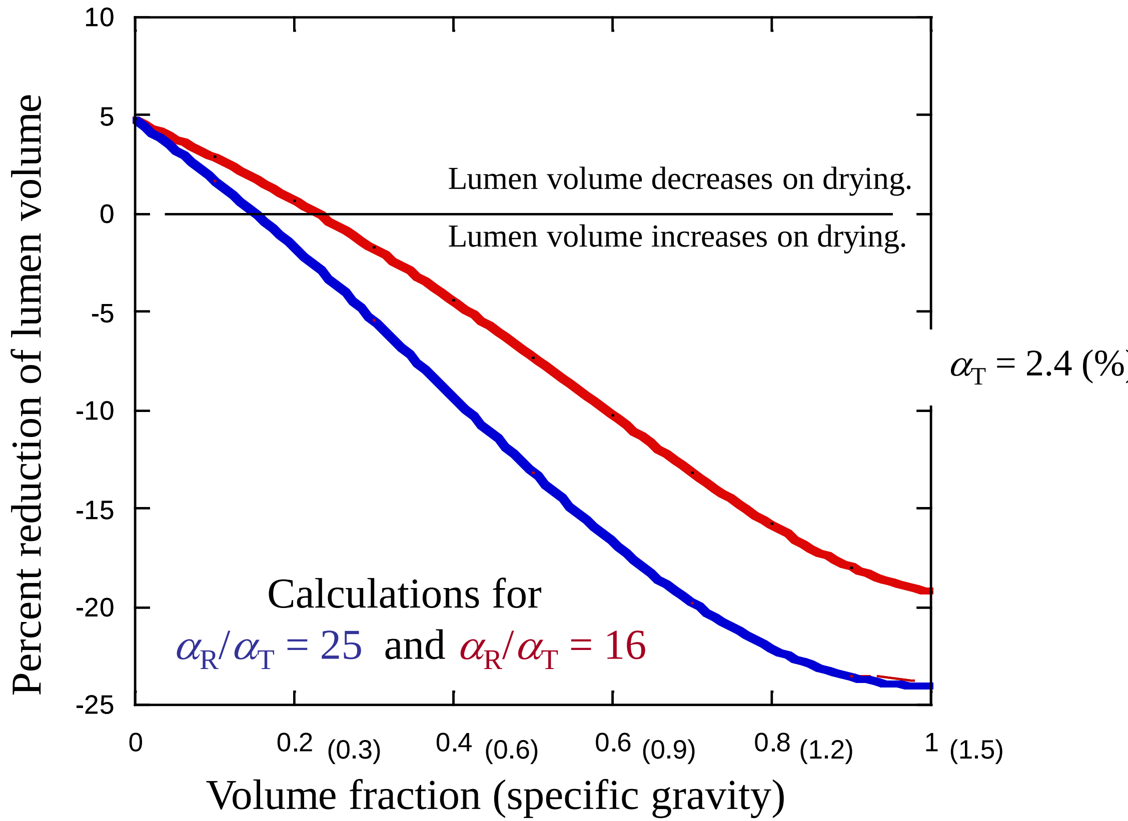
$$\frac{\Delta a}{a} = -\sqrt{\alpha_R \alpha_T}$$

Again, the same as before. **NO, NO, NO, wait a minute, the sign has changed!**
At some point when density increases, even though the wood shrinks the lumen diameter increases. Is this possible!?

Not only is it possible, it is exactly what happens.

For example, Boutelje* found that for Scots pine on drying, the lumen volume decreased in earlywood (thin cell walls) – as expected – but increased in latewood (thick cell walls) – not at all obvious. Similar observations were made by Beiser** for spruce and beech.

This is predicted by the current model.



*J. Boutelje, On the relationship between structure and the shrinkage and swelling of the wood in Swedish pine (*Pinus sylvestris*) and spruce (*Picea abies*). *Svensk Papperstidning* **76** (1973), 78-83.

** W.Beiser, Mikrophotgraphische Quellungsuntersuchungen von … . *Kolloid-Zeitschrift* **65** (1933), 203-211.

A Necessary Disclosure

After preparing this talk I came across a paper by T. Nakano* in which the density dependence of shrinkage/swelling is also considered via a cylindrical model. This model is essentially geometrical, not mechanical. It assumes that there exists a physical parameter for wood dependent on change of moisture content, $\Delta h/\Delta b$, [h is the cell wall thickness $(b-a)$]. The model results in a linear relationship between shrinkage/swelling and density – the slope being dependent on the value chosen for this physical parameter.

(In short, the model indirectly assumes a relationship between density and shrinkage in order to demonstrate a relationship between density and shrinkage.)

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*T. Nakano, Analysis of cell wall swelling on the basis of a cylindrical model. *Holzforschung* **62** (2008), 352-356.

~~High density woods have proportionately more cell wall and less lumen volume, and so shrink and swell more.~~

High density woods have proportionately more cell wall and less lumen volume; they shrink and swell more due to the unique nature of the microstructure.

End of Story

Thanks for listening.