Accuracy of modeling of bound water diffusion in wood induced by variation of ambient air parameters

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Introduction

Wooden objects are in permanent contact with moist air which is characterized by daily and seasonal variations of its parameters.

The parameter variation induces changes in bound water content of wood.

The water gain or loss is responsible for dimensional changes which lead to stress development and therefore to the risk of cracks and deformations.

Introduction

The accurate modeling of bound water transport is critical for assessing environmental risks to wooden cultural objects.

The quality of the modeling depends on

- transition between air parameters and wood equilibrium moisture content (EMC),
- structure of the diffusion model,
- values of coefficients of the model.

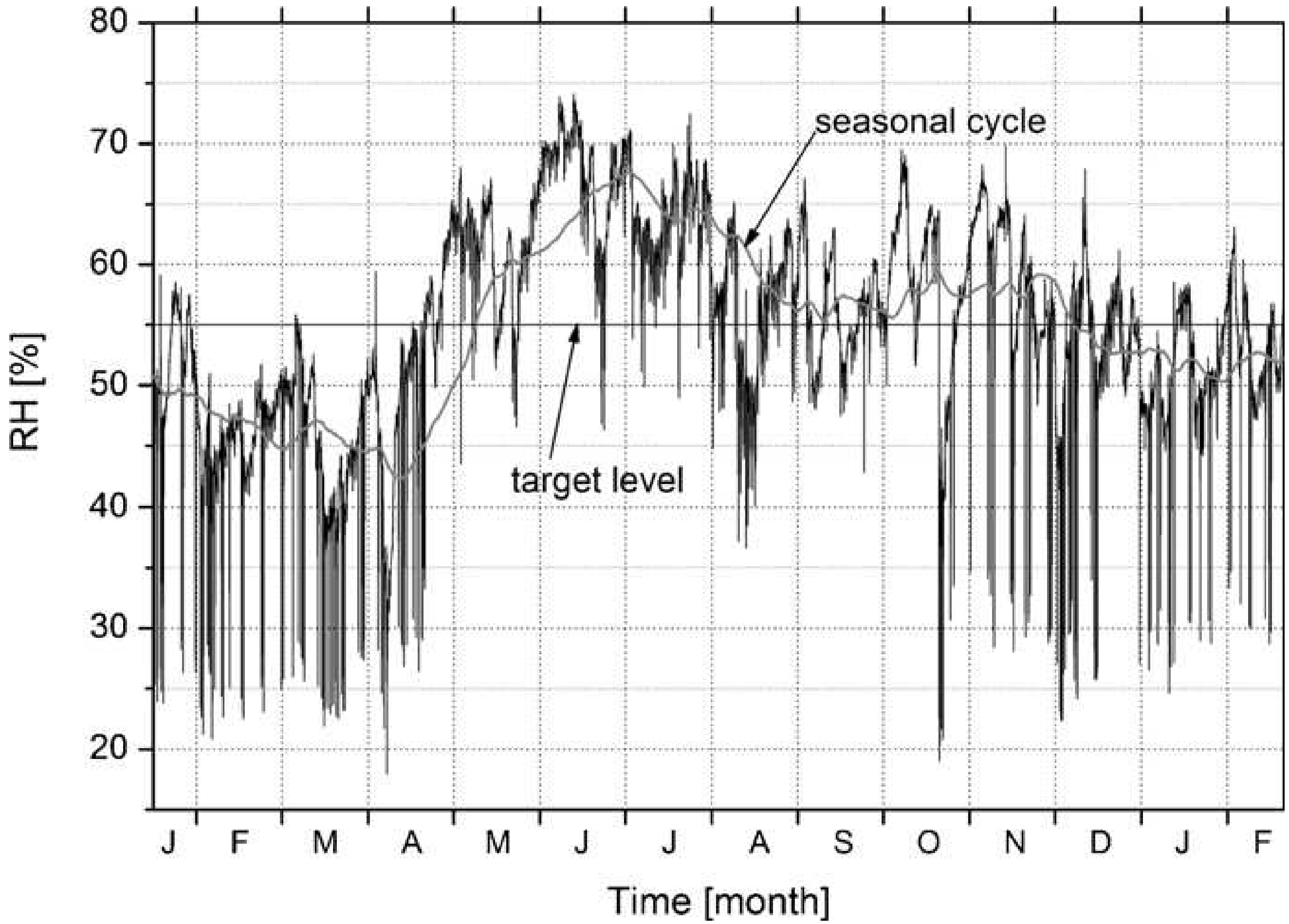
Introduction

The objective of the present study is to identify and assess factors influencing accuracy of modeling of the bound water diffusion induced by variations in the air parameters.

Transition between air parameters and wood EMC

Indoor RH variation

Santa Maria Maddalena church in Rocca Pietore



Taken from Camuffo D., Pagan E., Rissanen S., Bratasz Ł., Kozłowski R., Camuffo M., della Valle A. (2010): Journal of Cultural Heritage, 11: 205–219

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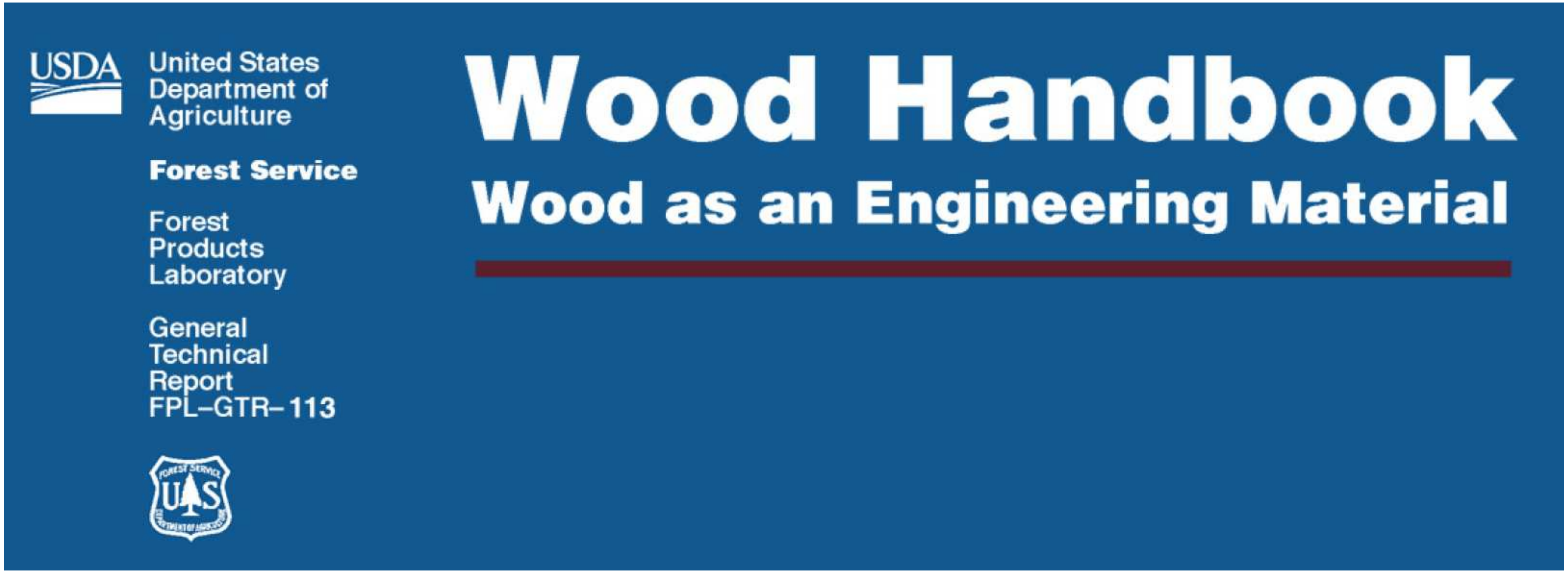
USDA United States Department of Agriculture

Forest Service

Forest Products Laboratory

General Technical Report FPL–GTR–113

Wood Handbook

Wood as an Engineering Material

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Table 3–4. Moisture content of wood in equilibrium with stated temperature and relative humidity

Temperature		Moisture content (%) at various relative humidity values																		
(°C	(°F))	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
–1.1	(30)	1.4	2.6	3.7	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.4	11.3	12.4	13.5	14.9	16.5	18.5	21.0	24.3
4.4	(40)	1.4	2.6	3.7	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.4	11.3	12.3	13.5	14.9	16.5	18.5	21.0	24.3
10.0	(50)	1.4	2.6	3.6	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.3	11.2	12.3	13.4	14.8	16.4	18.4	20.9	24.3
15.6	(60)	1.3	2.5	3.6	4.6	5.4	6.2	7.0	7.8	8.6	9.4	10.2	11.1	12.1	13.3	14.6	16.2	18.2	20.7	24.1
21.1	(70)	1.3	2.5	3.5	4.5	5.4	6.2	6.9	7.7	8.5	9.2	10.1	11.0	12.0	13.1	14.4	16.0	17.9	20.5	23.9
26.7	(80)	1.3	2.4	3.5	4.4	5.3	6.1	6.8	7.6	8.3	9.1	9.9	10.8	11.7	12.9	14.2	15.7	17.7	20.2	23.6
32.2	(90)	1.2	2.3	3.4	4.3	5.1	5.9	6.7	7.4	8.1	8.9	9.7	10.5	11.5	12.6	13.9	15.4	17.3	19.8	23.3
37.8	(100)	1.2	2.3	3.3	4.2	5.0	5.8	6.5	7.2	7.9	8.7	9.5	10.3	11.2	12.3	13.6	15.1	17.0	19.5	22.9
43.3	(110)	1.1	2.2	3.2	4.0	4.9	5.6	6.3	7.0	7.7	8.4	9.2	10.0	11.0	12.0	13.2	14.7	16.6	19.1	22.4
48.9	(120)	1.1	2.1	3.0	3.9	4.7	5.4	6.1	6.8	7.5	8.2	8.9	9.7	10.6	11.7	12.9	14.4	16.2	18.6	22.0
54.4	(130)	1.0	2.0	2.9	3.7	4.5	5.2	5.9	6.6	7.2	7.9	8.7	9.4	10.3	11.3	12.5	14.0	15.8	18.2	21.5
60.0	(140)	0.9	1.9	2.8	3.6	4.3	5.0	5.7	6.3	7.0	7.7	8.4	9.1	10.0	11.0	12.1	13.6	15.3	17.7	21.0
65.6	(150)	0.9	1.8	2.6	3.4	4.1	4.8	5.5	6.1	6.7	7.4	8.1	8.8	9.7	10.6	11.8	13.1	14.9	17.2	20.4
71.1	(160)	0.8	1.6	2.4	3.2	3.9	4.6	5.2	5.8	6.4	7.1	7.8	8.5	9.3	10.3	11.4	12.7	14.4	16.7	19.9
76.7	(170)	0.7	1.5	2.3	3.0	3.7	4.3	4.9	5.6	6.2	6.8	7.4	8.2	9.0	9.9	11.0	12.3	14.0	16.2	19.3
82.2	(180)	0.7	1.4	2.1	2.8	3.5	4.1	4.7	5.3	5.9	6.5	7.1	7.8	8.6	9.5	10.5	11.8	13.5	15.7	18.7
87.8	(190)	0.6	1.3	1.9	2.6	3.2	3.8	4.4	5.0	5.5	6.1	6.8	7.5	8.2	9.1	10.1	11.4	13.0	15.1	18.1
93.3	(200)	0.5	1.1	1.7	2.4	3.0	3.5	4.1	4.6	5.2	5.8	6.4	7.1	7.8	8.7	9.7	10.9	12.5	14.6	17.5
98.9	(210)	0.5	1.0	1.6	2.1	2.7	3.2	3.8	4.3	4.9	5.4	6.0	6.7	7.4	8.3	9.2	10.4	12.0	14.0	16.9
104.4	(220)	0.4	0.9	1.4	1.9	2.4	2.9	3.4	3.9	4.5	5.0	5.6	6.3	7.0	7.8	8.8	9.9			
110.0	(230)	0.3	0.8	1.2	1.6	2.1	2.6	3.1	3.6	4.2	4.7	5.3	6.0	6.7						
115.6	(240)	0.3	0.6	0.9	1.3	1.7	2.1	2.6	3.1	3.5	4.1	4.6								
121.1	(250)	0.2	0.4	0.7	1.0	1.3	1.7	2.1	2.5	2.9										
126.7	(260)	0.2	0.3	0.5	0.7	0.9	1.1	1.4												
132.2	(270)	0.1	0.1	0.2	0.3	0.4	0.4													

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Parameterized by Hailwood-Horrobin model (two hydrates model)

$$M = \frac{1800}{W}\left[\frac{Kh}{1-Kh} + \frac{K_1Kh + 2K_1K_2K^2h^2}{1 + K_1Kh + K_1K_2K^2h^2}\right]$$

M (%) Equilibrium Moisture Content (EMC)

h (–) air relative humidity (RH)

W, K, K_1, K_2 – functions of temperature

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”The data do not distinguish between adsorption and desorption.”

Simpson W.T. (1973): Wood and Fiber, 5(1): 41-49.

”The data are based largely in Sitka spruce (*Picea sitchensis*)…”

Simpson W.T., Rosen H.N. (1981): Wood and Fiber, 13(3): 150-158.

The set of EMC values comes from different experiments originally reported in the 1930s.

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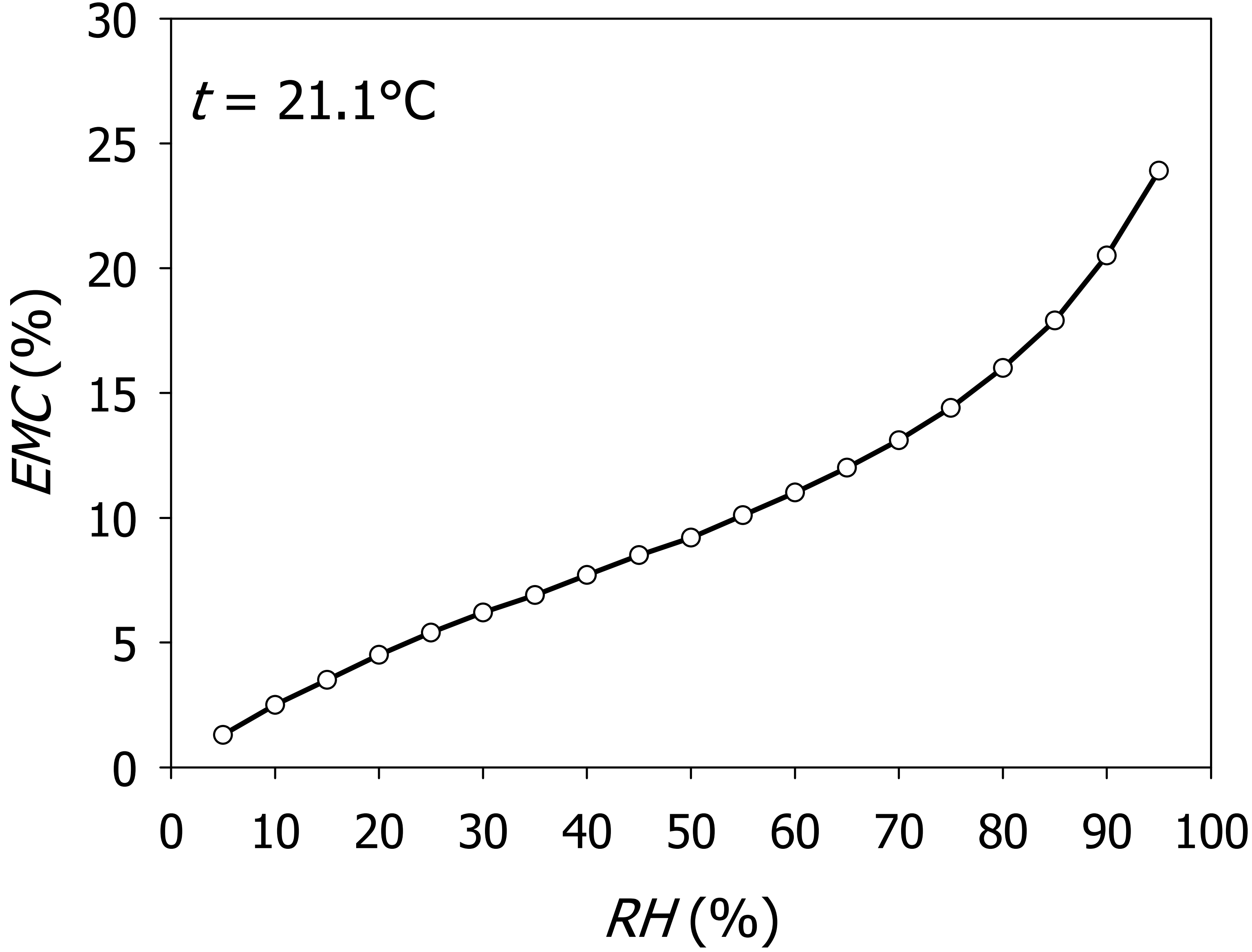
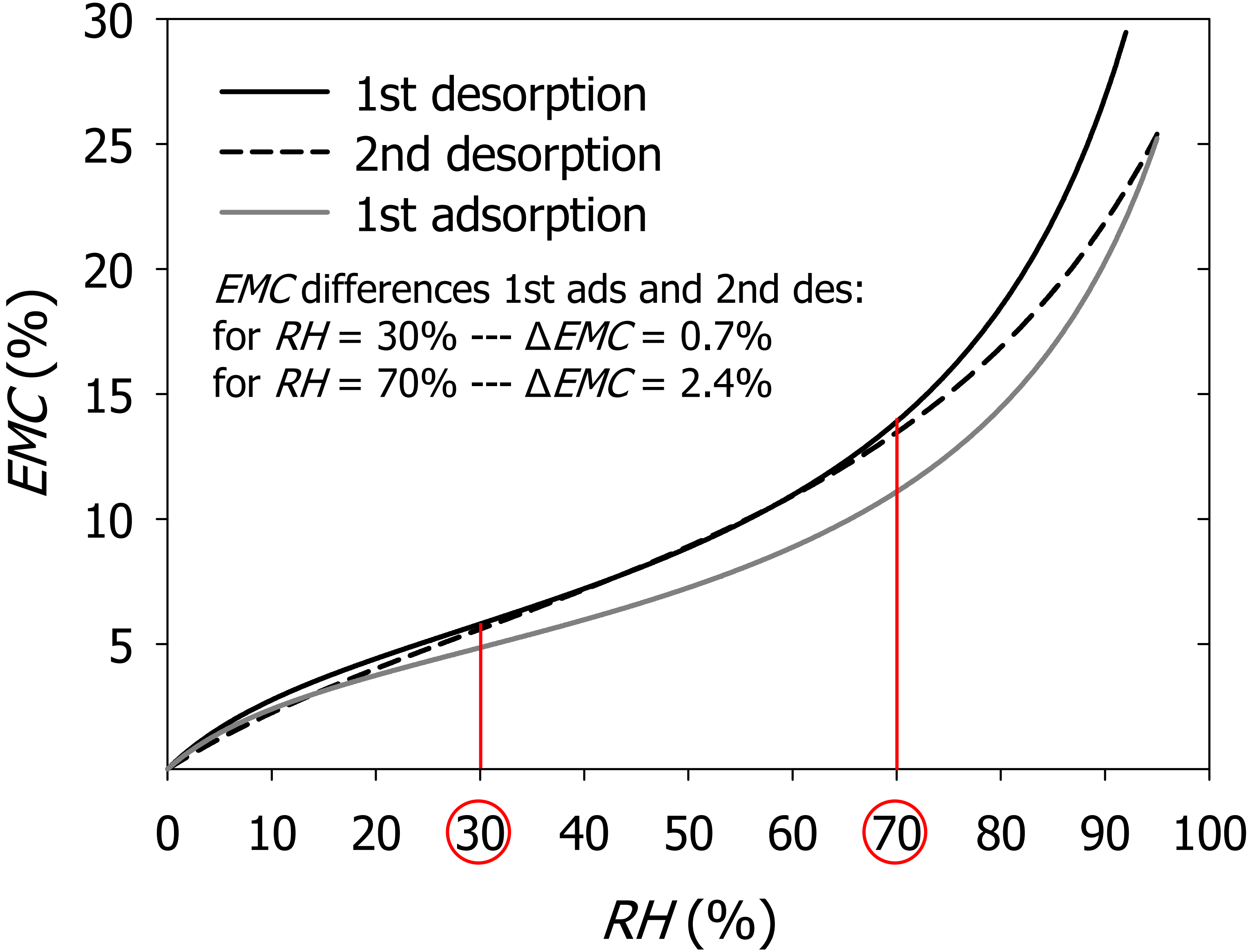


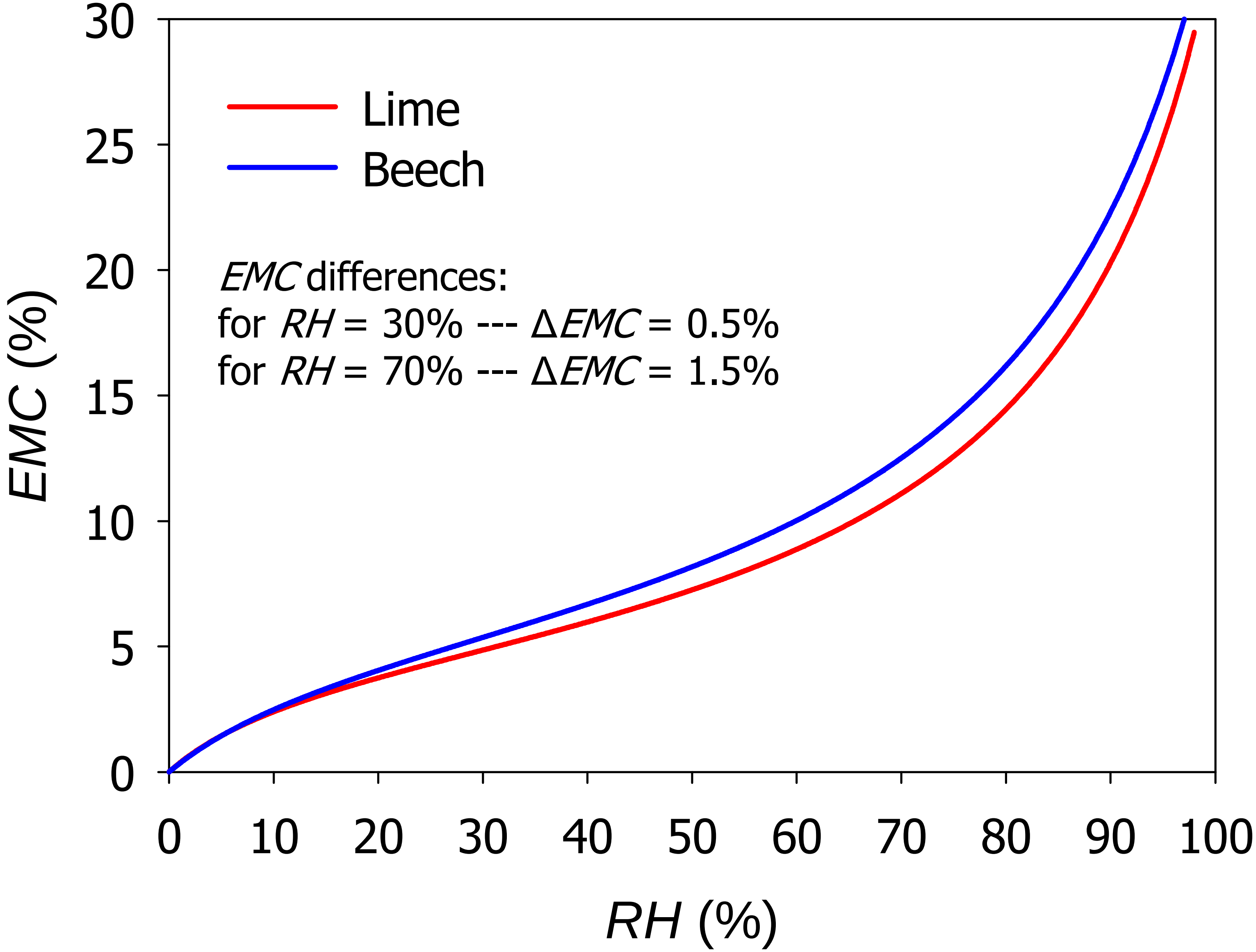
Sorption hysteresis

Mature lime wood



Wood species

Adsorption lime & beech



Structure of sorption models

3 parameters models

One hydrate Hailwood-Horrobin model

$$M = \frac{1800}{W}\left[\frac{Kh}{1 - Kh} + \frac{K_1 Kh}{1 + K_1 Kh}\right]$$

GAB model

$$M = m_0 \frac{ckh}{(1 - kh)(1 - kh + ckh)}$$

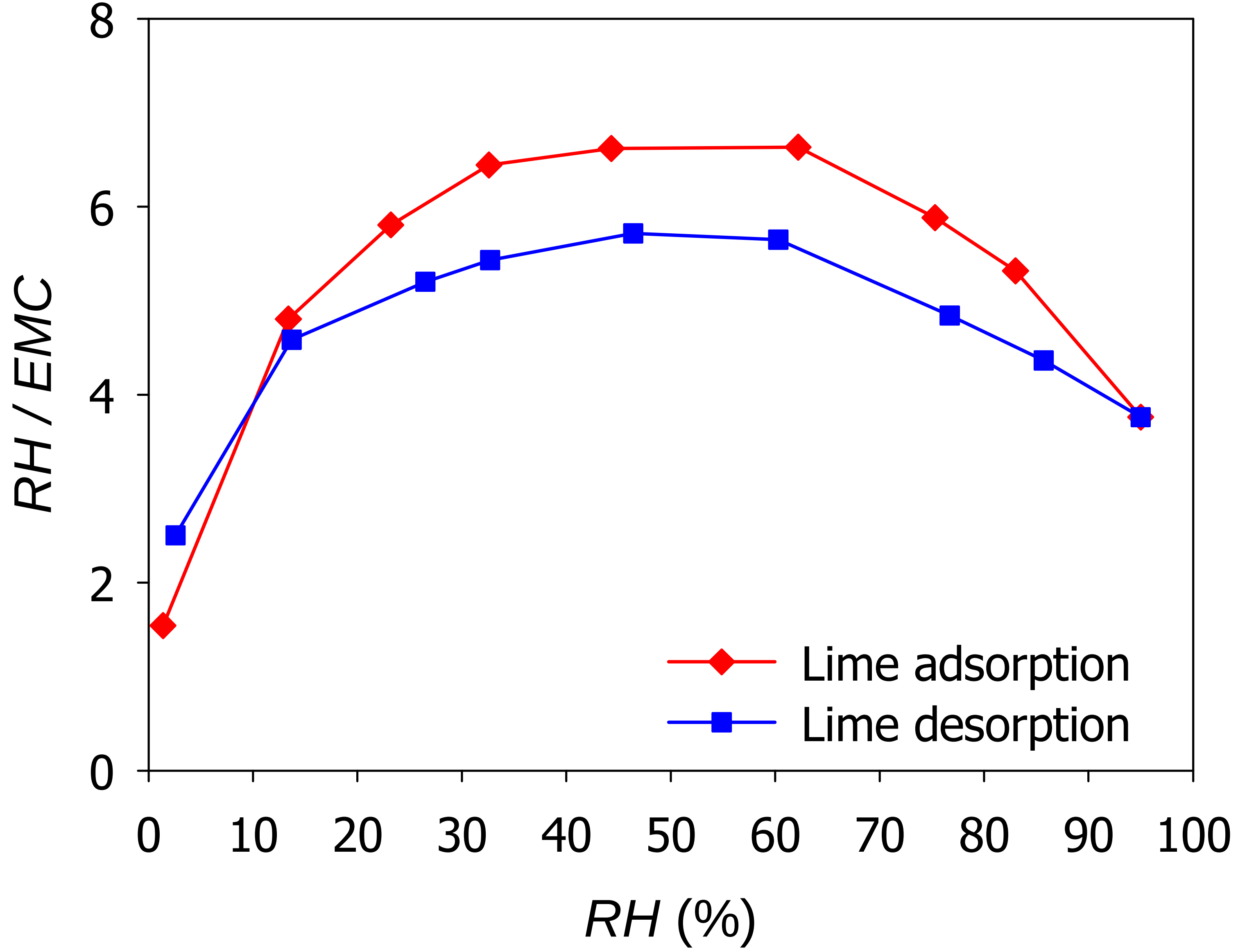
3 parameters models

The models can be reduced to the following form

$$\frac{h}{M} = A + Bh - Ch^2$$

Nakano (2006) showed that h / M = f(h) is not symmetric for the FPL data.

3 parameters models



Boundary conditions

Boundary conditions

The 1st kind b. condition = instant hygroscopic equilibrium at wood surface.

$$M(x,t) = M_\infty, \qquad (x,t) \in \Gamma \times [0, t_F]$$

The 3rd kind b. condition = transport resistance between air and wood surface.

$$\left(-D\frac{\partial M}{\partial x}\right) = \sigma \cdot \left[M(x,t) - M_\infty\right], \qquad (x,t) \in \Gamma \times [0, t_F]$$

Boundary conditions

Modified 3rd kind b. condition =
transport resistance between air and wood surface
+ exponential approach to EMC at wood surface.

$$\left(-D\frac{\partial M}{\partial x}\right)=\sigma\cdot\left[M(x,t)-M_{\infty}\right],\qquad (x,t)\in\Gamma\times\left[0,t_F\right]$$

$$M_{\infty}=a+b\cdot\left[1-\exp(-t/c)\right]$$

Methods for diffusion coefficient determination

Analytical methods

Initial sorption

$$D = \frac{\pi}{4/l^2}\left(\frac{\Delta E}{\Delta\sqrt{t}}\right)^2$$

Final sorption

$$D = \frac{4l^2}{\pi^2 t}\ln\left\{\frac{8}{\pi^2(1-E)}\right\}$$

Assumptions:

- the wood surface obtains instant hygroscopic equilibrium with moist air,
- the diffusion coefficient is a constant (it does not depend on water content).

Analytical methods

Separation of the diffusion coefficient and the surface emission factor

Choong & Skaar (1969)

$$\frac{Dt_1}{l_1^2} = 0.2 + 0.7\frac{D}{\sigma l_1}$$

$$\frac{Dt_2}{l_2^2} = 0.2 + 0.7\frac{D}{\sigma l_2}$$

(Liu 1989)

$$D = l^2 \frac{-0.1654}{0.701\frac{dt}{dE} + 2.05t}$$

$$\sigma = \frac{0.701D}{D\frac{t}{l} - 0.1963l}$$

Assumption:

- the diffusion coefficient is a constant (it does not depend on water content).

Analytical methods

The analytical methods require information on EMC:

- from sorption isotherms, or
- from direct measurements during sorption experiments.

Inverse method

Model of bound water diffusion = partial differential equation + initial and boundary conditions.

Results from sorption experiments = bound water content in time.

Software for finding model coefficients and equilibrium water content = solving diffusion model and optimizing objective function.

Inverse method

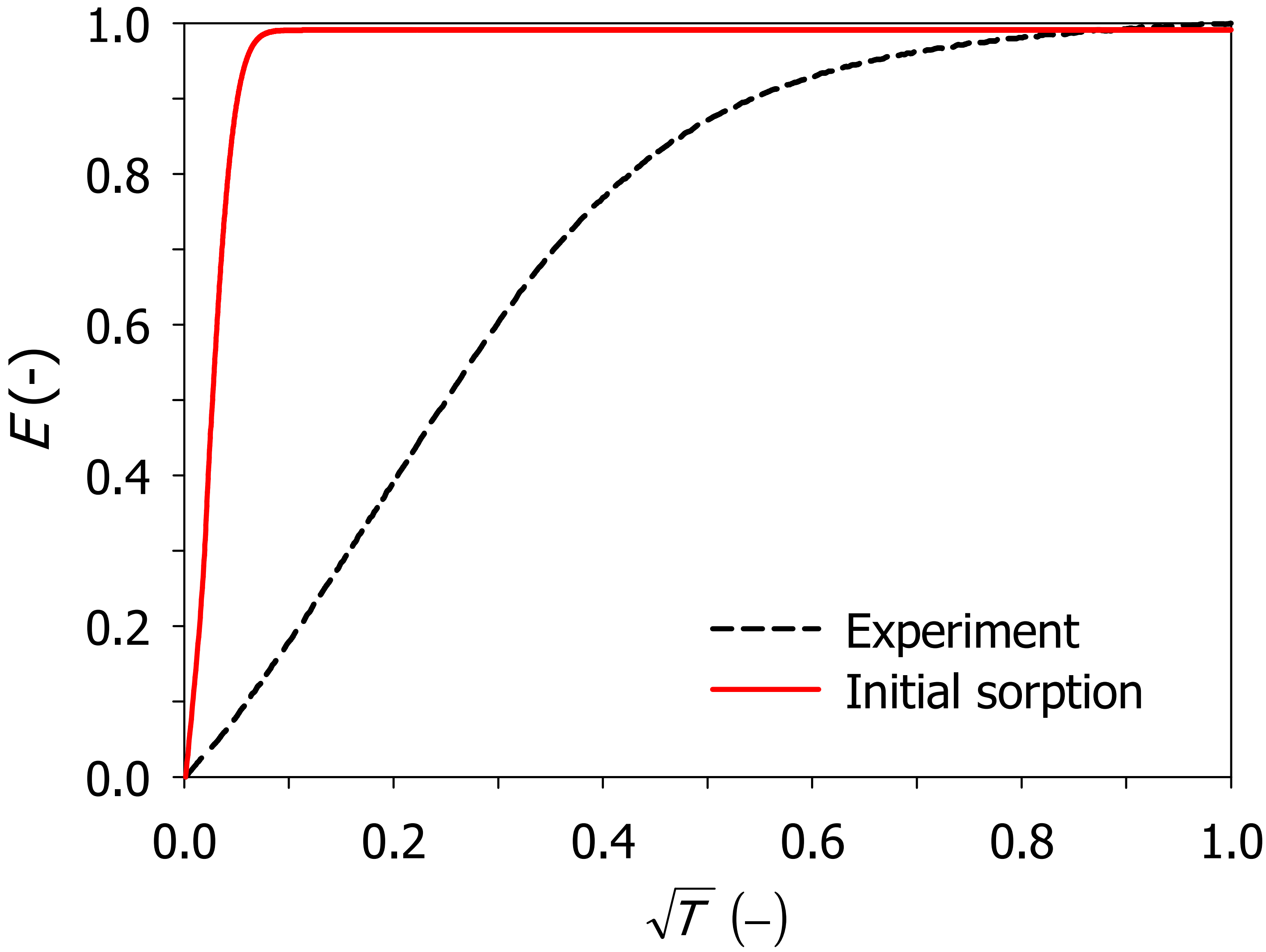
Diffusion coefficient values dependent on bound water content and parameterized by functions.

The boundary conditions of any kind.

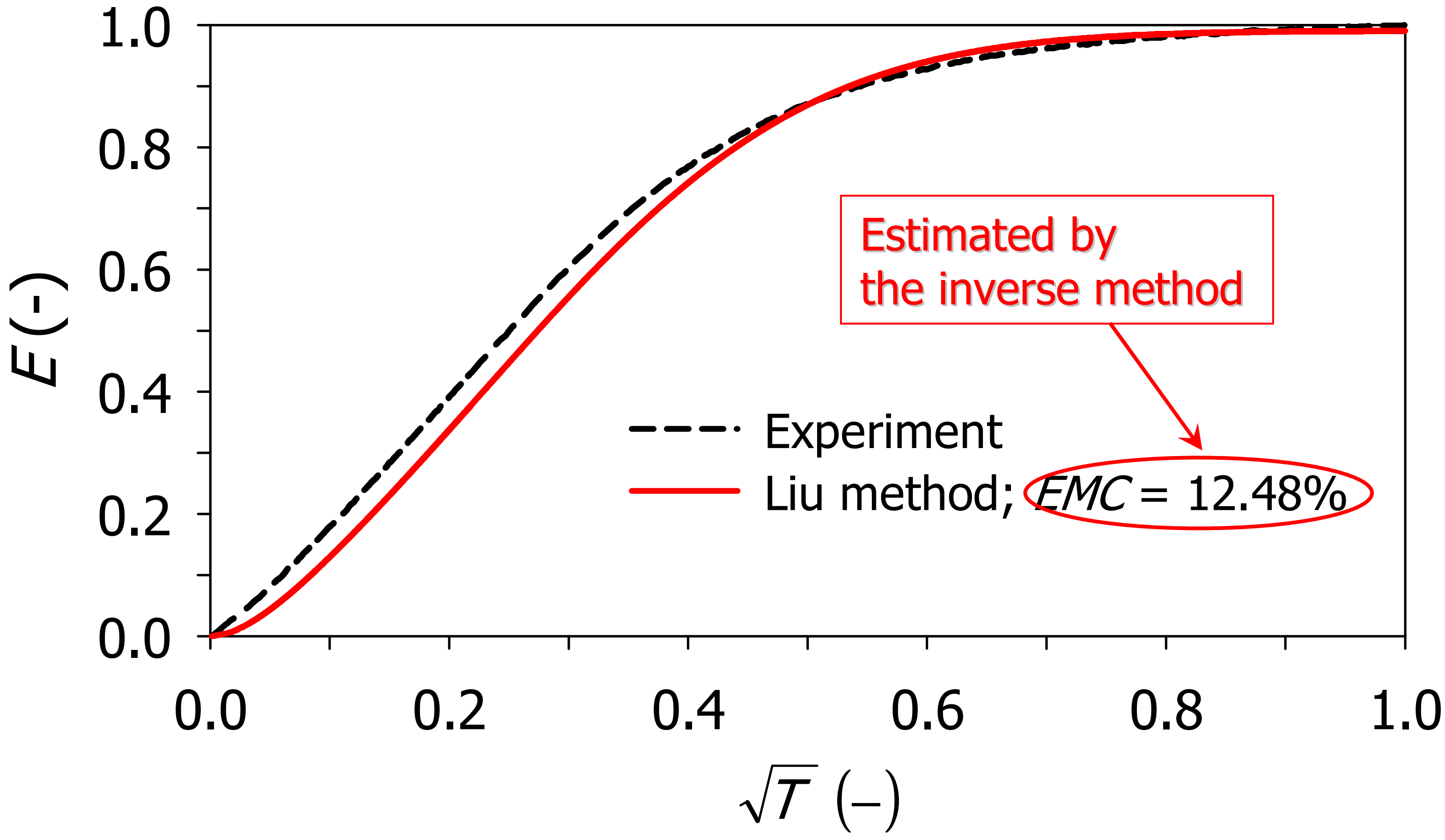
Equilibrium water content is not measured but estimated.

Results of modeling

Initial sorption method

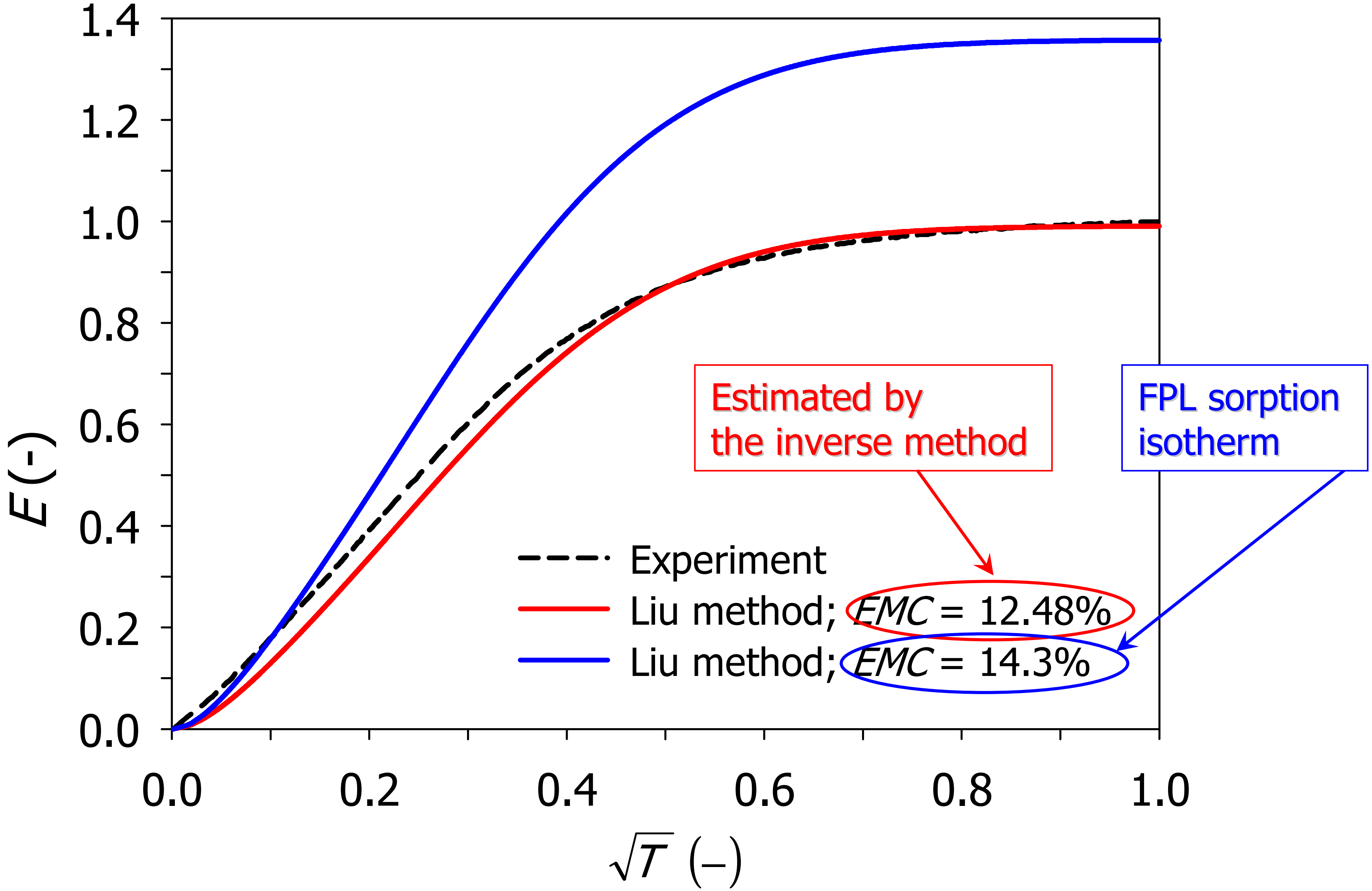


Liu method

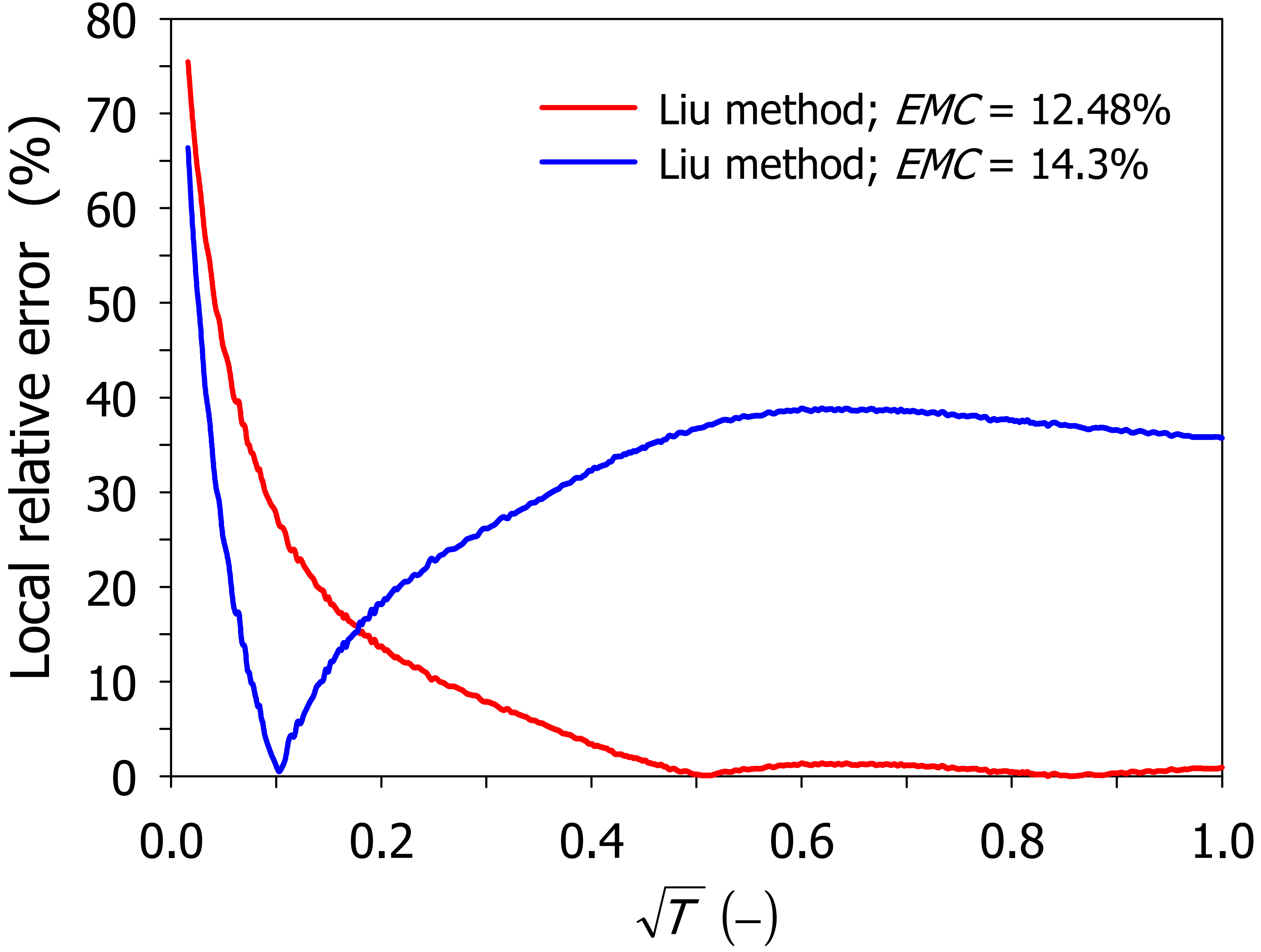


1.0
0.8
0.6
0.4
0.2
0.0
E (-)
0.0
0.2
0.4
0.6
0.8
1.0
$\sqrt{T}$ (–)
Estimated by
the inverse method
Experiment
Liu method; EMC = 12.48%

Liu method

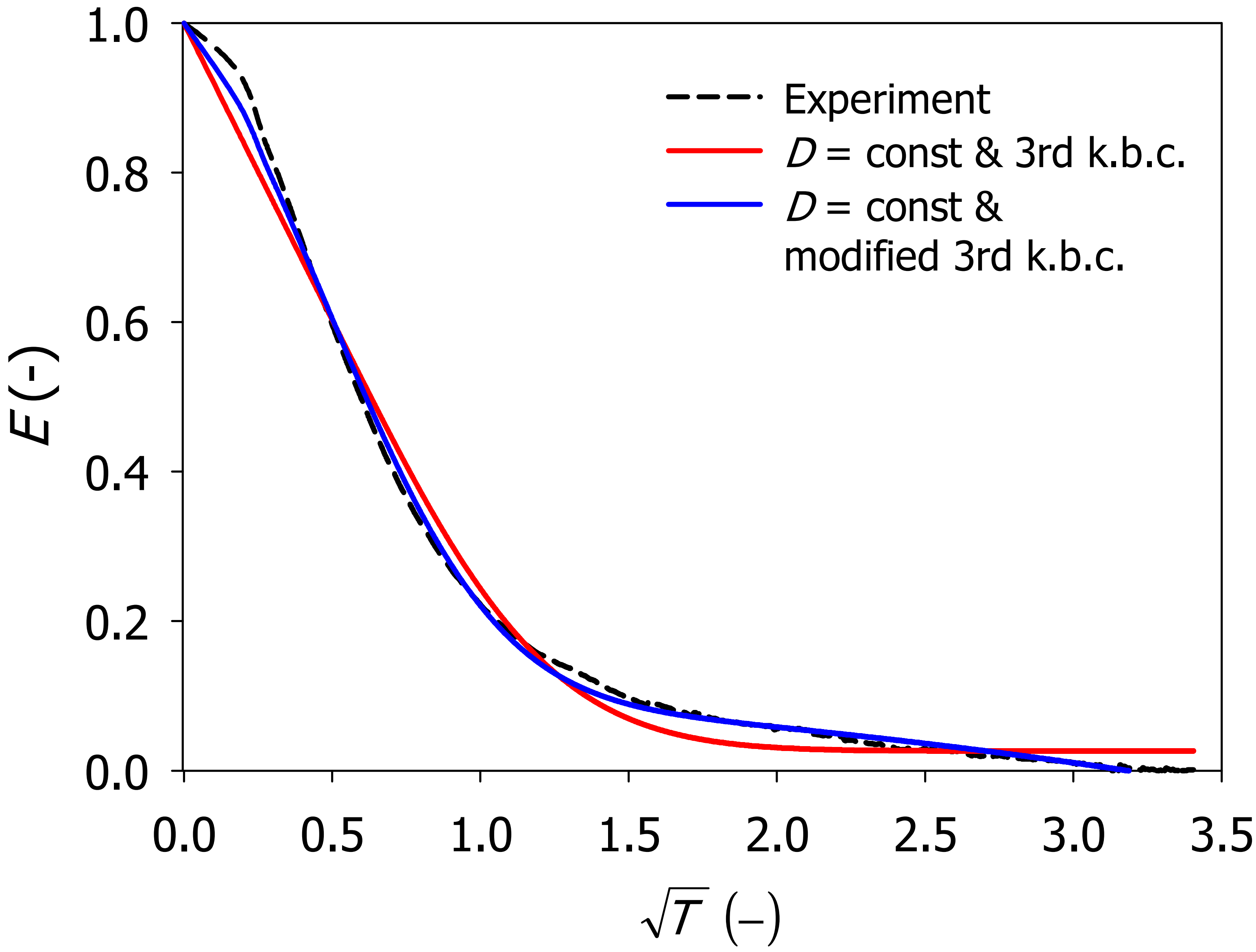

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
E (-)
0.0
0.2
0.4
0.6
0.8
1.0
√T (–)
Estimated by
the inverse method
FPL sorption
isotherm
Experiment
Liu method; EMC = 12.48%
Liu method; EMC = 14.3%

Liu method



Liu method; *EMC* = 12.48%
Liu method; *EMC* = 14.3%
Local relative error (%)
$\sqrt{T}$ (–)
0
10
20
30
40
50
60
70
80
0.0
0.2
0.4
0.6
0.8
1.0

Inverse method



Concluding remarks

1. The application of the sorption isotherms should account at least for wood species, hysteresis, structure of sorption models.

2. The first kind boundary condition is not acceptable for the diffusion modeling in wood.

3. Accuracy of the diffusion modeling is mostly affected not only by the third kind boundary condition but also by its modified form.