Materials Science & Technology

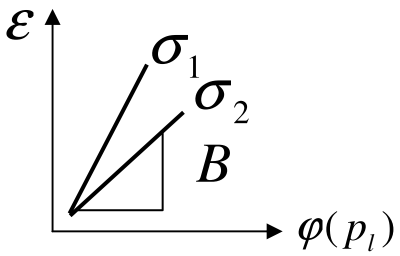
Poroelastic modelling of the coupled mechanical moisture behaviour of wood

M. Dressler, D. Derome, R. Guyer and J. Carmeliet

1. objective

known effects of water

- swelling

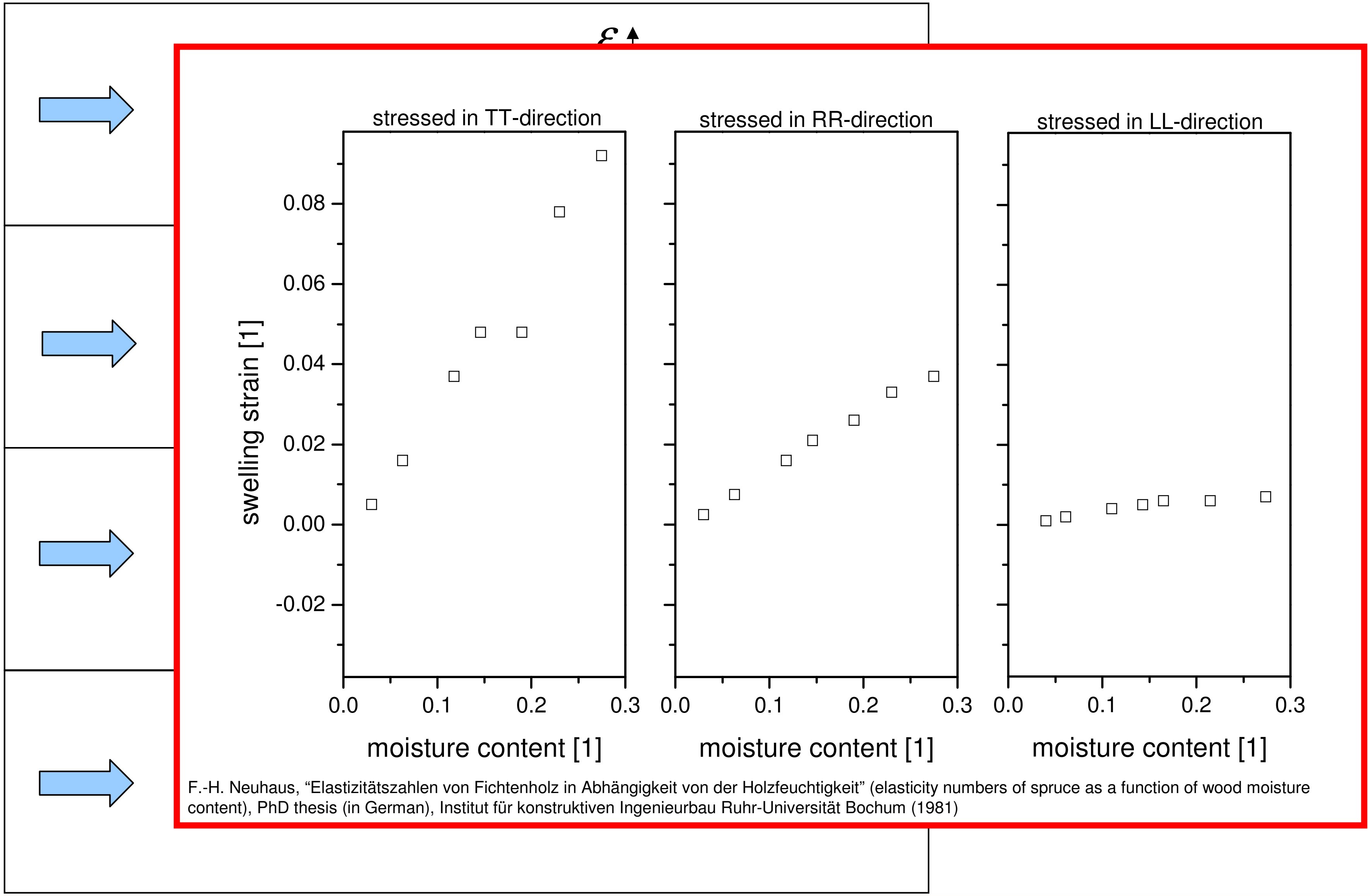


- modulus of elasticity
- Poisson‘s ratio

- water sorption

- stress dependent u

known effects of water



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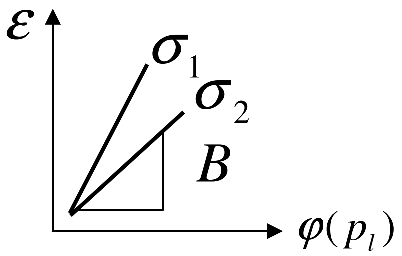

- modulus of elasticity
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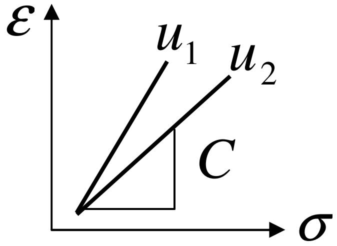

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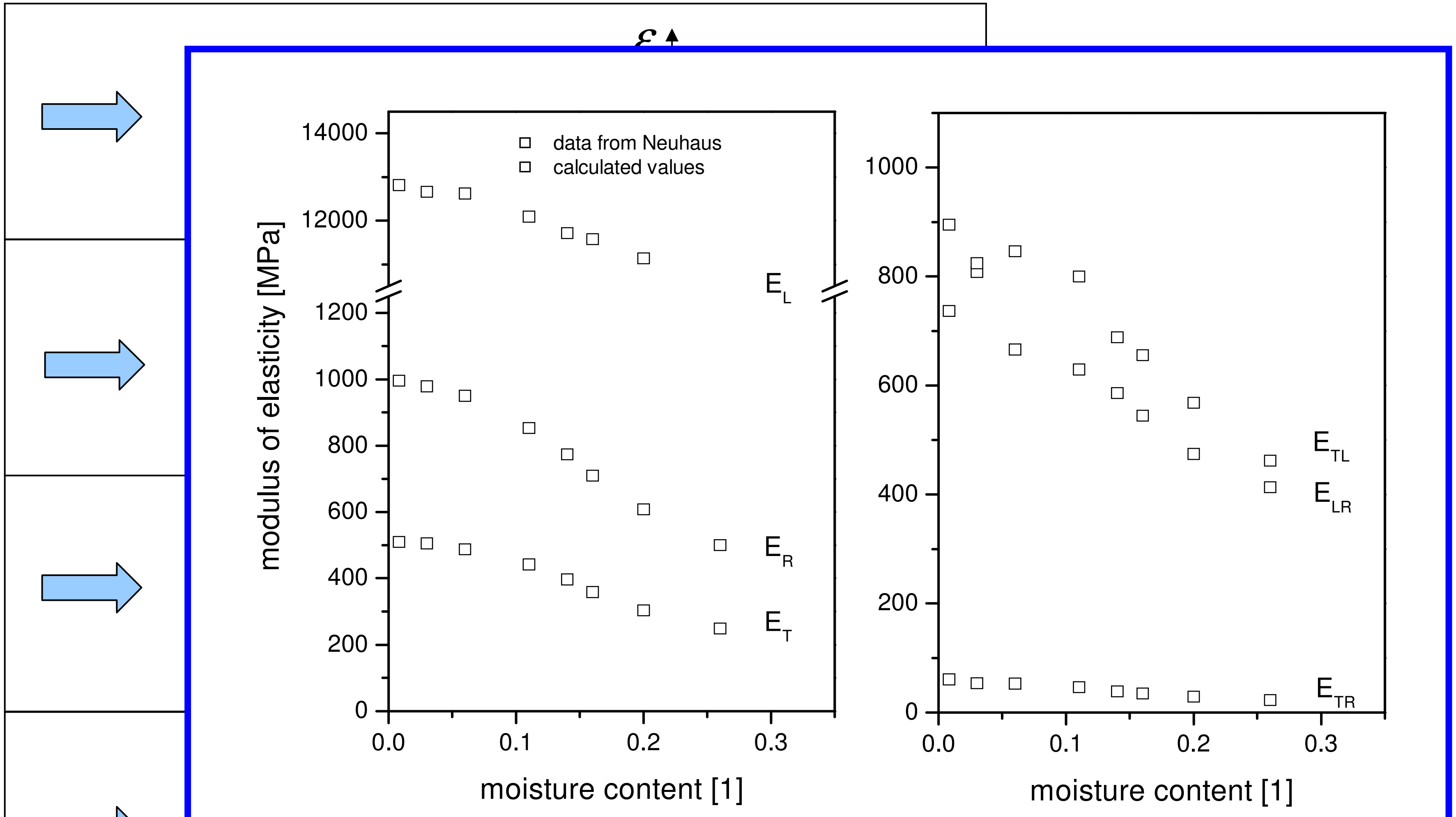

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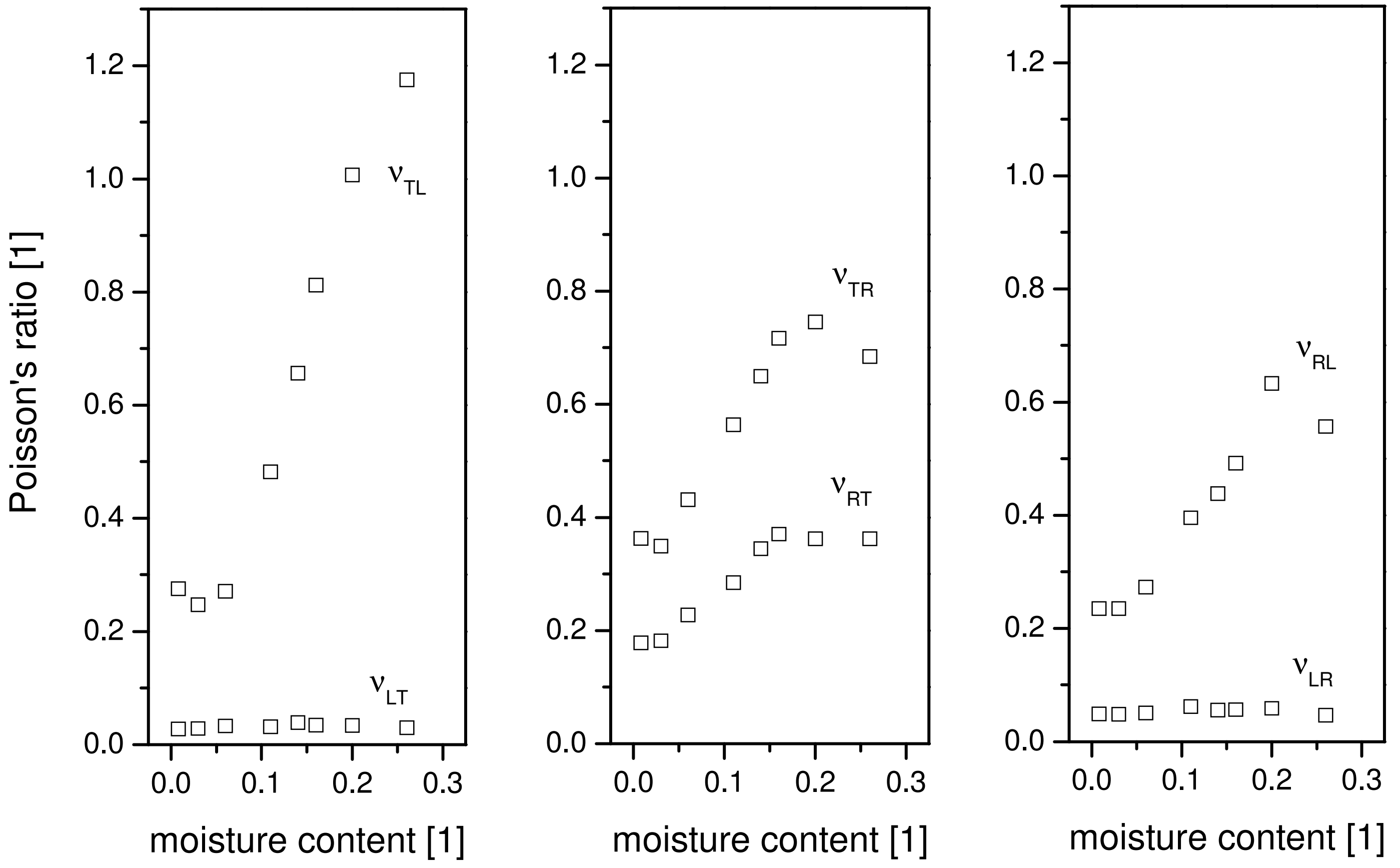

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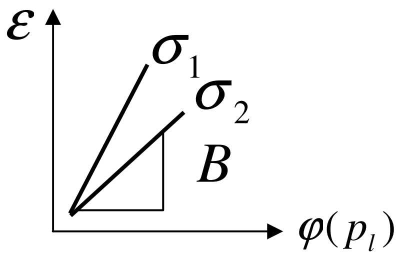
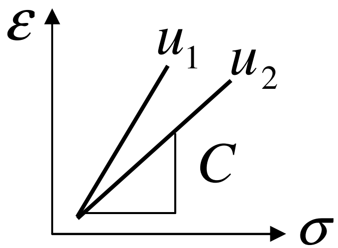
constitutive equations

- swelling



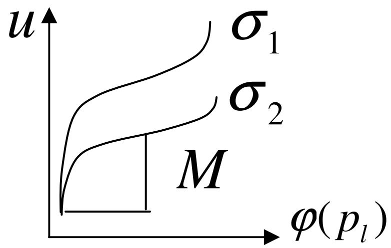
$$d\varepsilon = B(\sigma)d\varphi$$

- modulus of elasticity
- Poisson‘s ratio



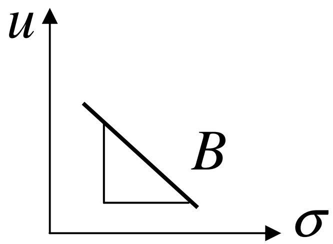
$$d\varepsilon = C(u)d\sigma$$

- water sorption



$$du = M(\sigma)d\varphi$$

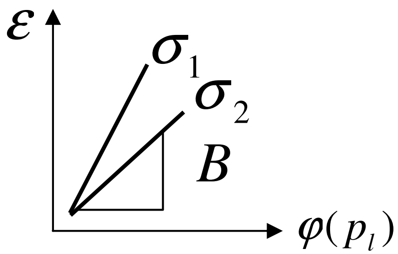
- stress dependent u



$$du = B(\varphi)d\sigma$$

constitutive equations

- swelling



- modulus of elasticity
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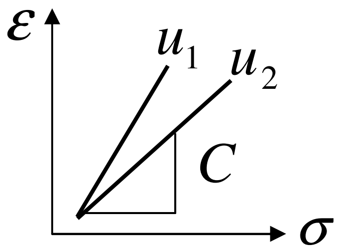
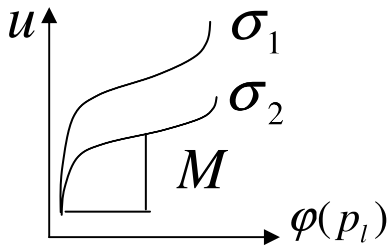


- water sorption



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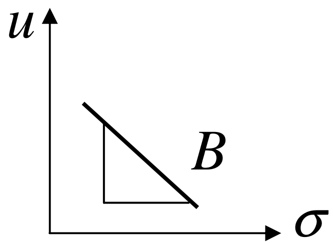


$$d\varepsilon = B(\sigma)d\varphi$$

$$d\varepsilon = C(u)d\sigma$$

$$d\varepsilon = Cd\sigma + Bdp_l$$

$$du = M(\sigma)d\varphi$$

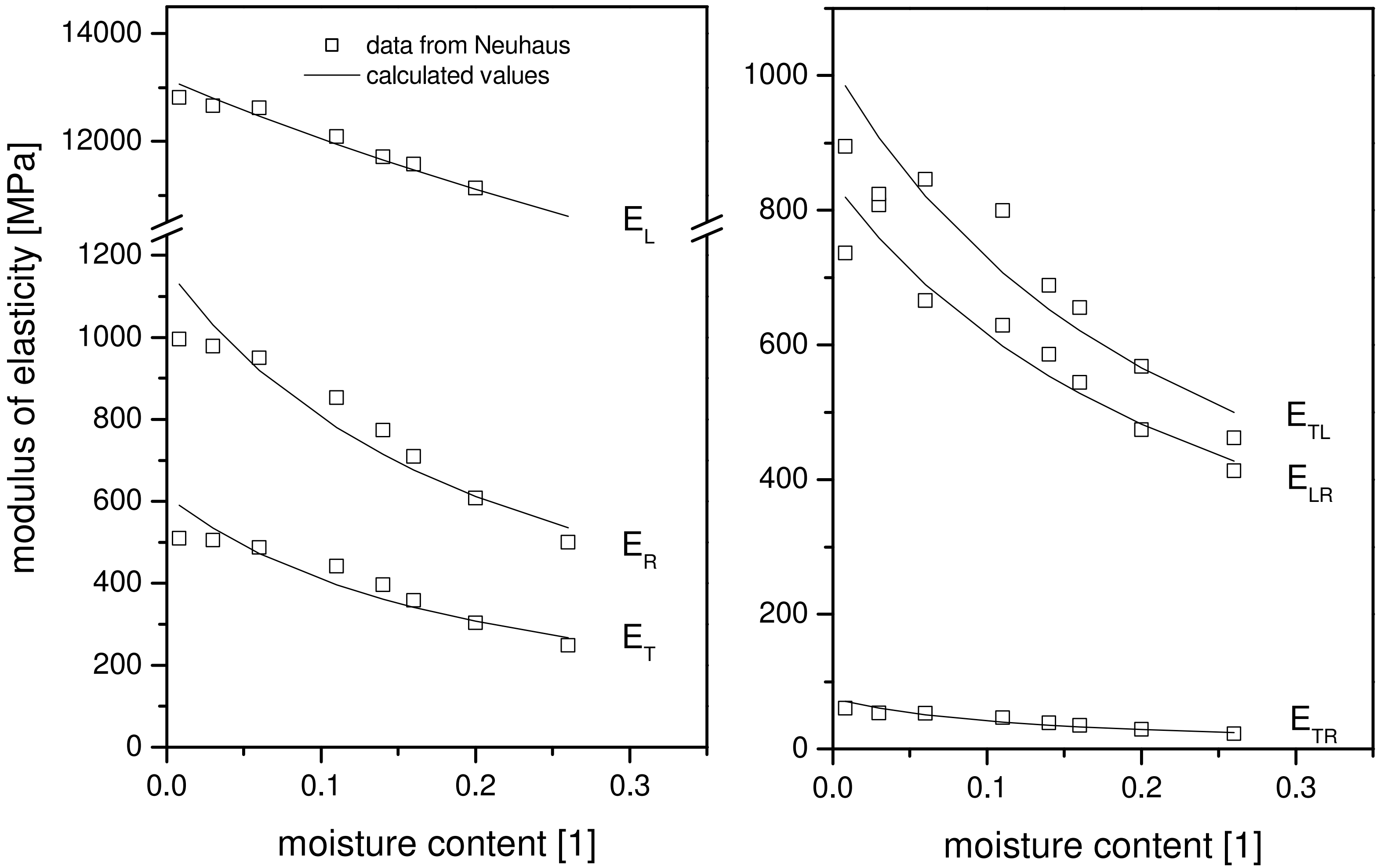
$$du = B(\varphi)d\sigma$$

$$du = Mdp_l + Bd\sigma$$

J. Carmeliet, R. Guyer, D. Derome, 6'th Plant Biomechanics Conference, 2009

2. application

results



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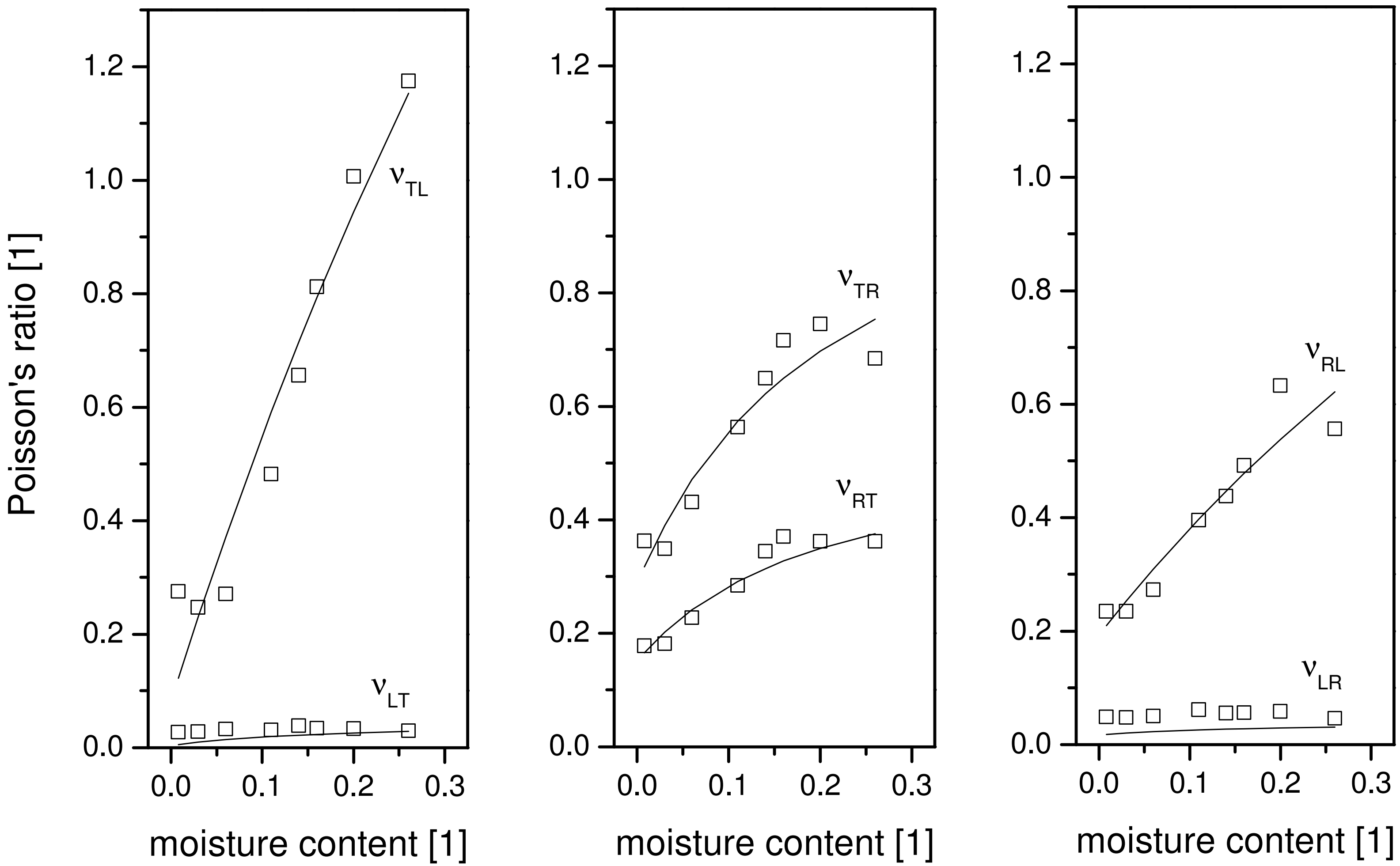
results



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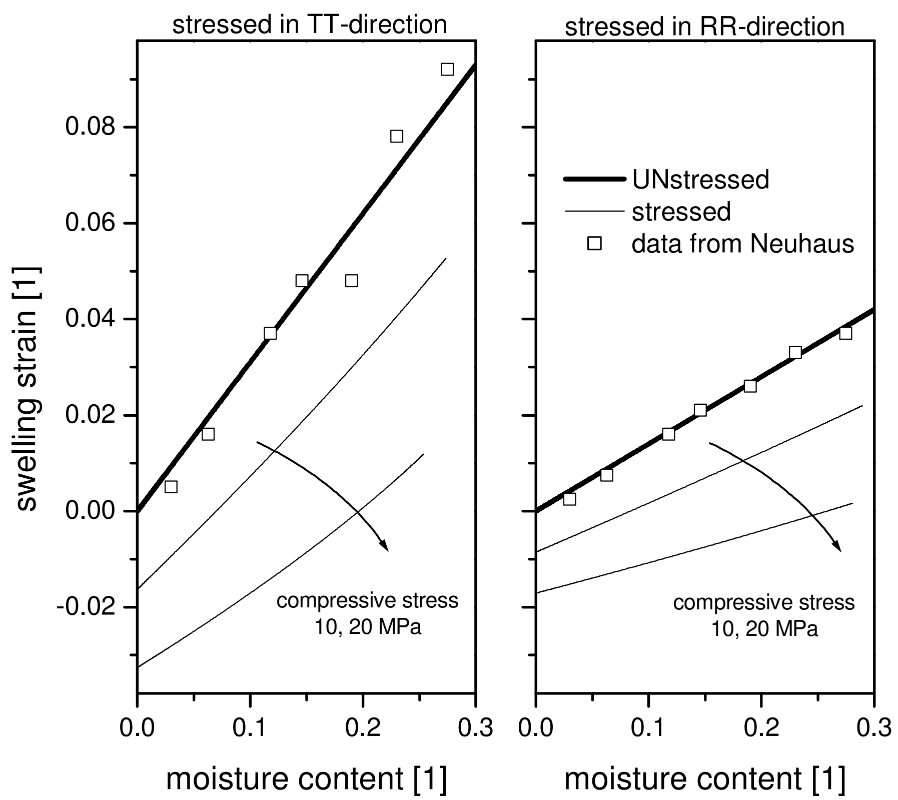

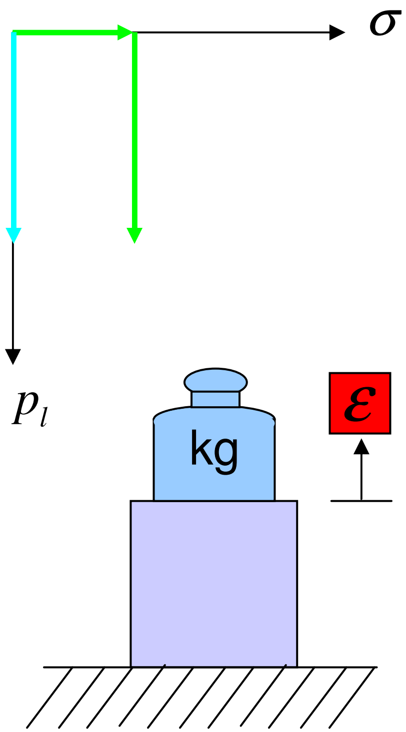

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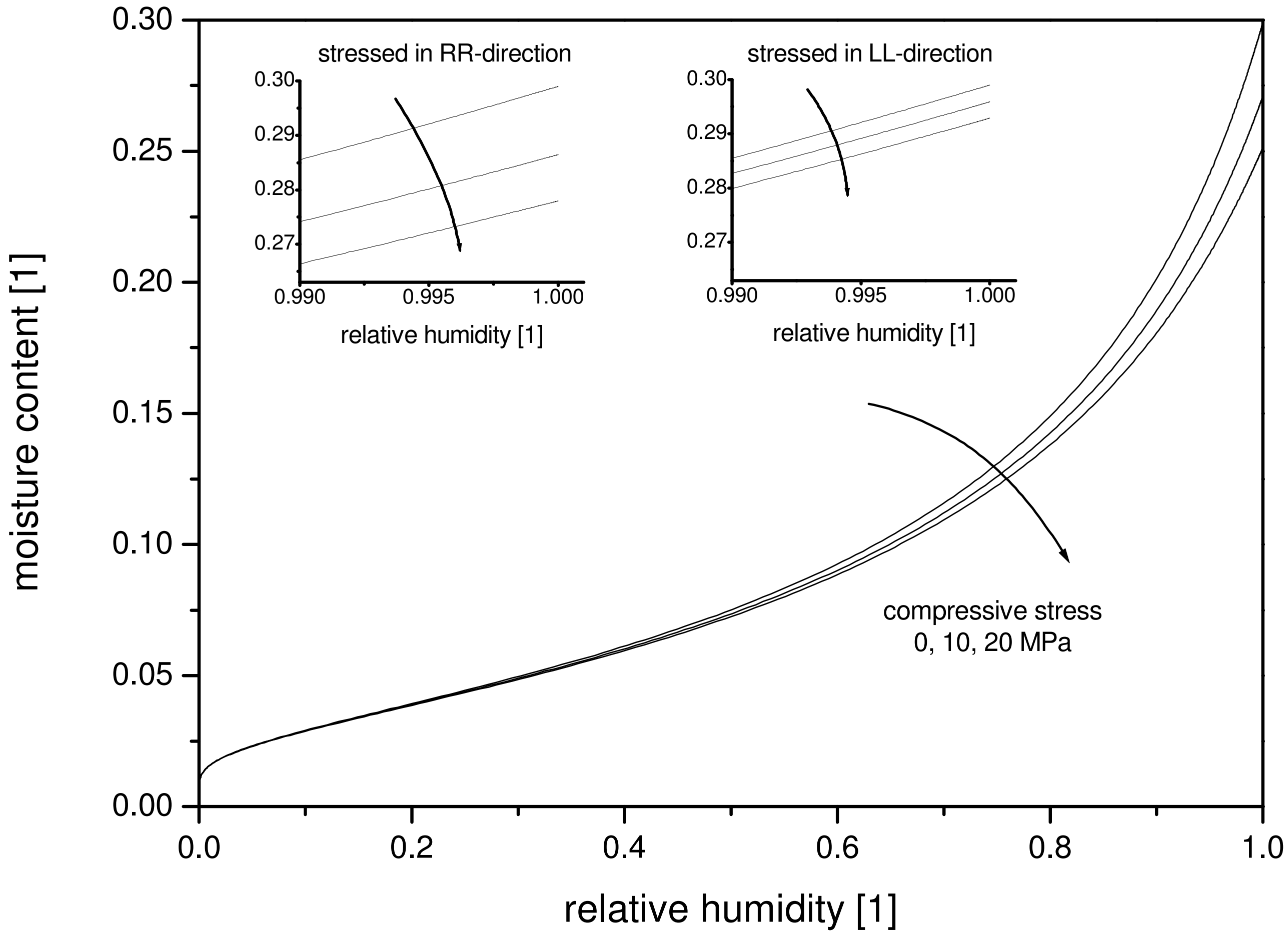
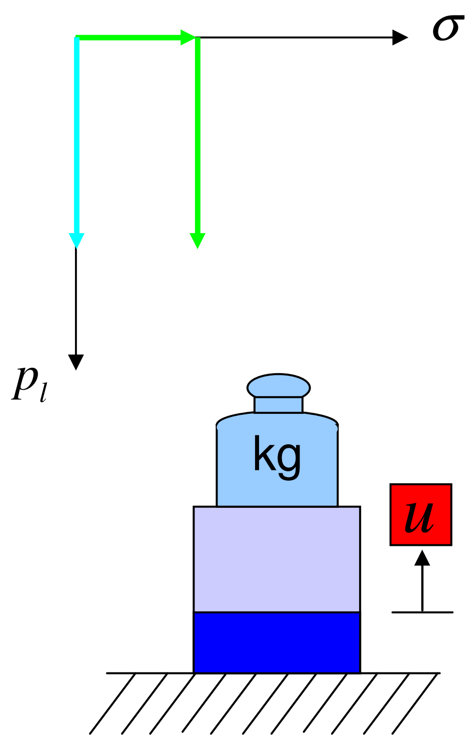
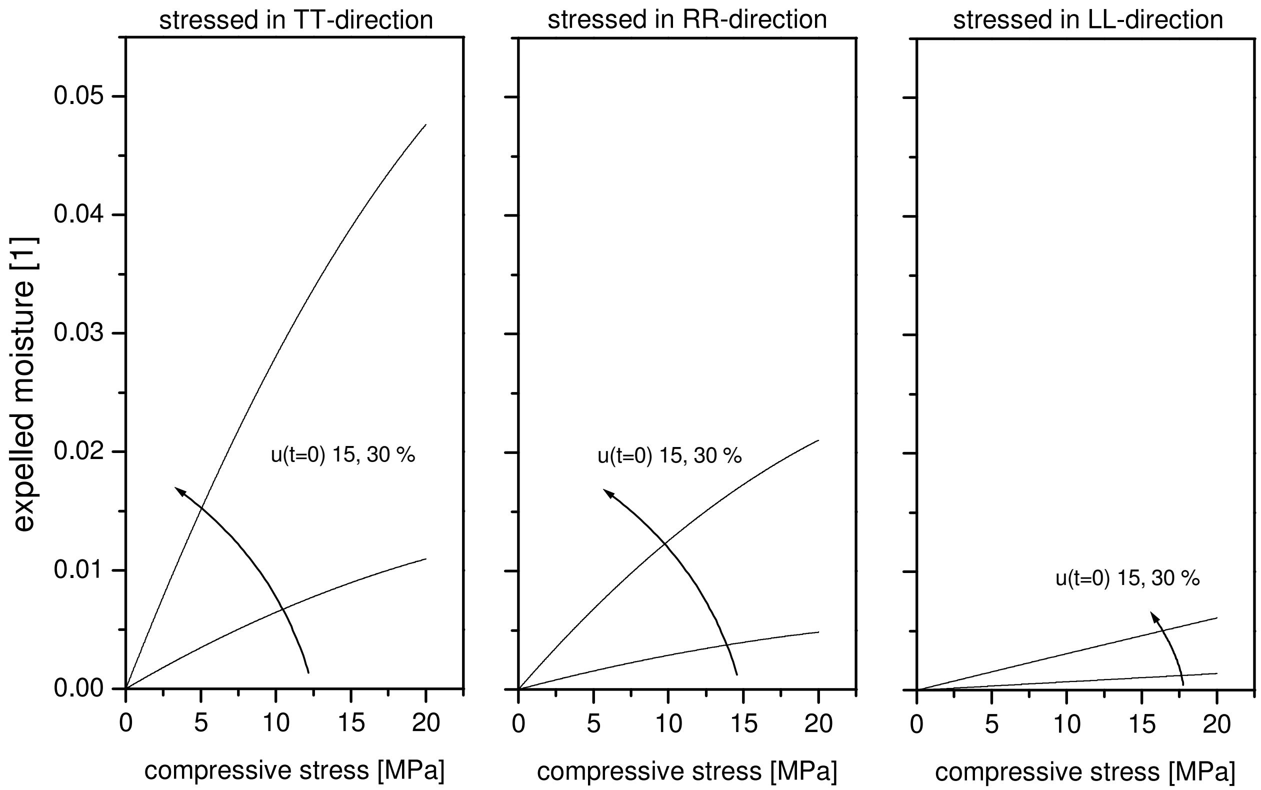
results

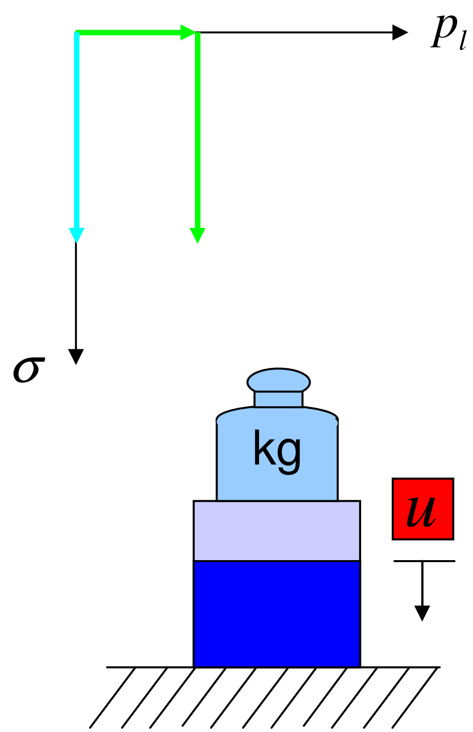

stressed in RR-direction
0.30
0.29
0.28
0.27
0.990
0.995
1.000
relative humidity [1]
stressed in LL-direction
0.30
0.29
0.28
0.27
0.990
0.995
1.000
relative humidity [1]
moisture content [1]
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
relative humidity [1]
compressive stress
0, 10, 20 MPa


σ
p_l
kg
u

results

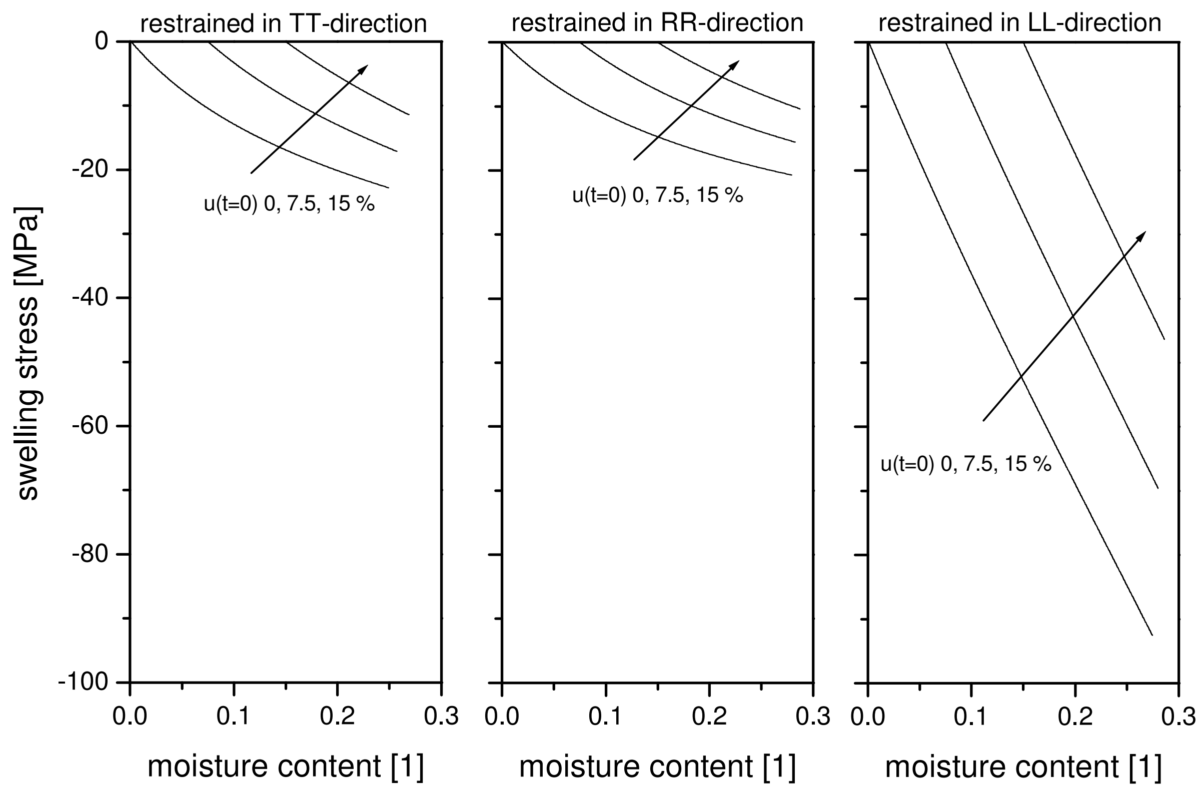


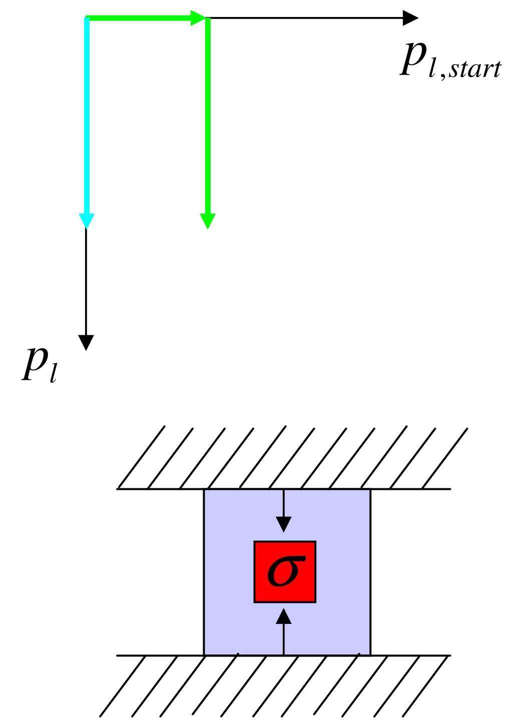
stressed in TT-direction
stressed in RR-direction
stressed in LL-direction
expelled moisture [1]
0.05
0.04
0.03
0.02
0.01
0.00
0
5
10
15
20
compressive stress [MPa]
u(t=0) 15, 30 %



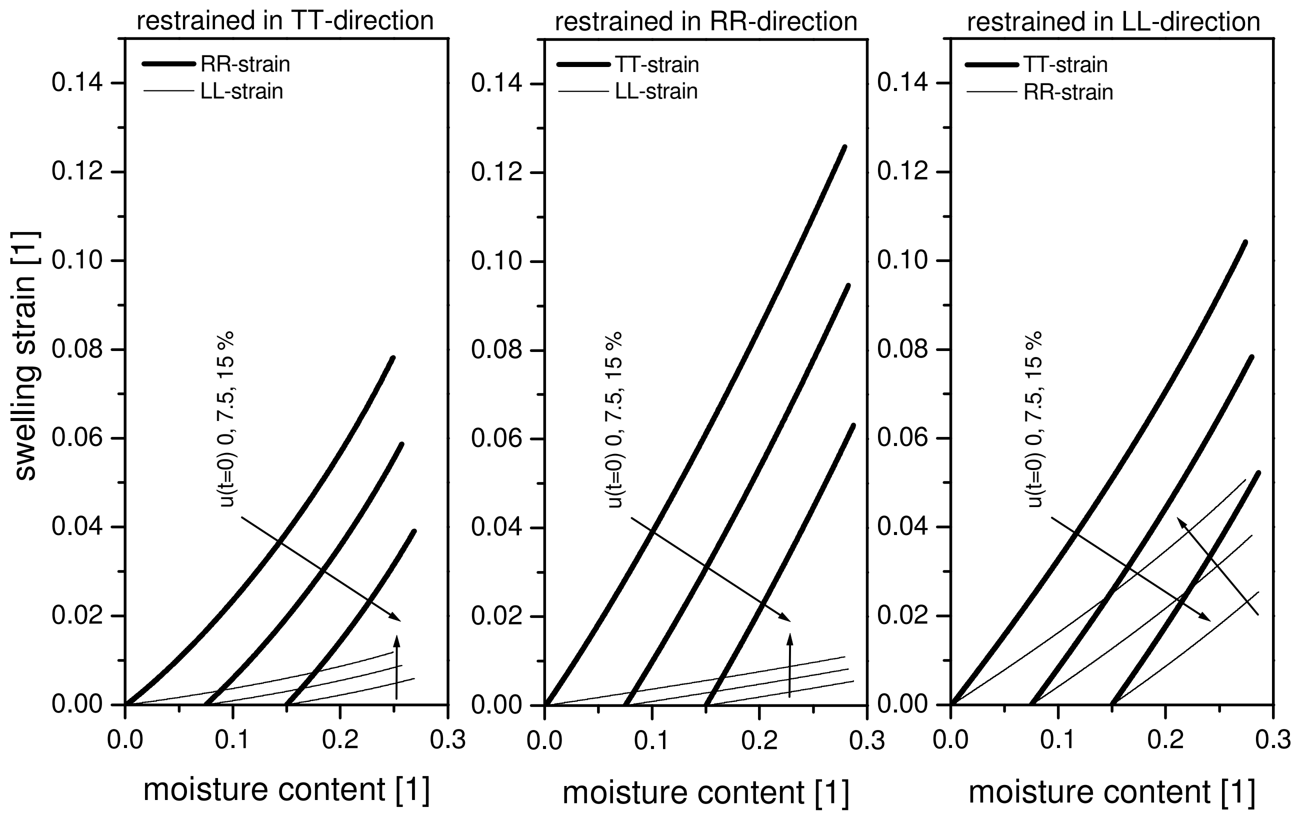
p_l
σ
kg
u

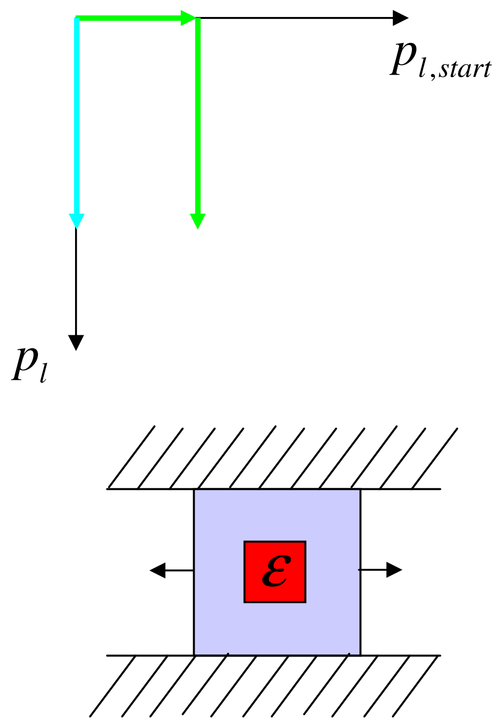
results


restrained in TT-direction
restrained in RR-direction
restrained in LL-direction
swelling stress [MPa]
0
-20
-40
-60
-80
-100
u(t=0) 0, 7.5, 15 %
u(t=0) 0, 7.5, 15 %
u(t=0) 0, 7.5, 15 %
0.0
0.1
0.2
0.3
moisture content [1]
moisture content [1]
moisture content [1]


$p_{l,start}$
p_l
σ

results

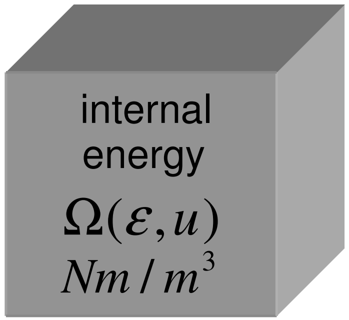

restrained in TT-direction
RR-strain
LL-strain
restrained in RR-direction
TT-strain
LL-strain
restrained in LL-direction
TT-strain
RR-strain
swelling strain [1]
u(t=0) 0, 7.5, 15 %
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.0
0.1
0.2
0.3
moisture content [1]


$p_{l,start}$
p_l
ε

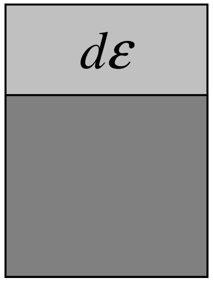
3. physical background

coupled moisture and mechanical behavior

1. total differential $d\Omega(\varepsilon,u)$ $$d\Omega(\varepsilon,u) = \frac{\partial \Omega}{\partial \varepsilon} d\varepsilon + \frac{\partial \Omega}{\partial u} du$$

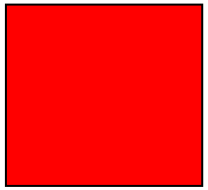


$d\Omega \propto d\varepsilon$



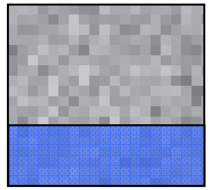
$d\Omega \propto dT$

dT



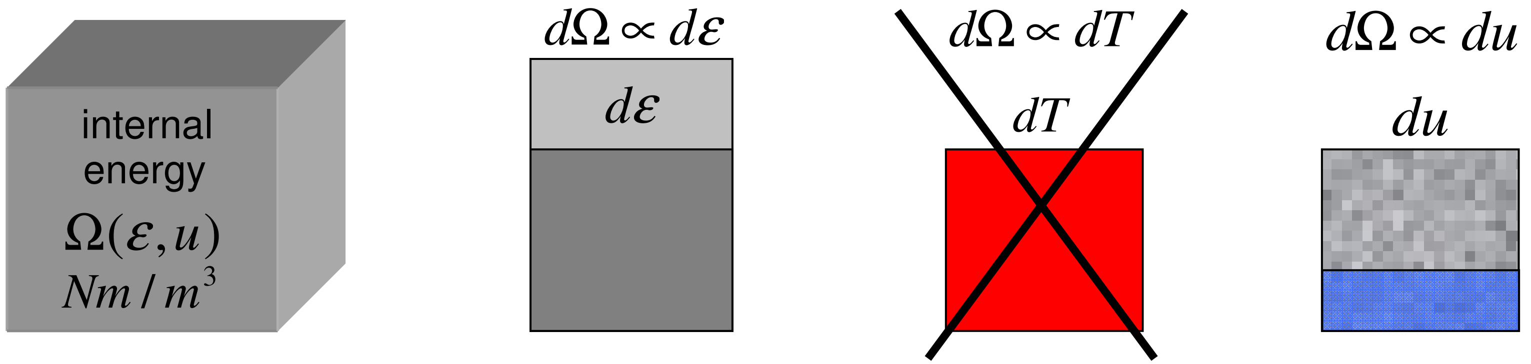
$d\Omega \propto du$

du



coupled moisture and mechanical behavior

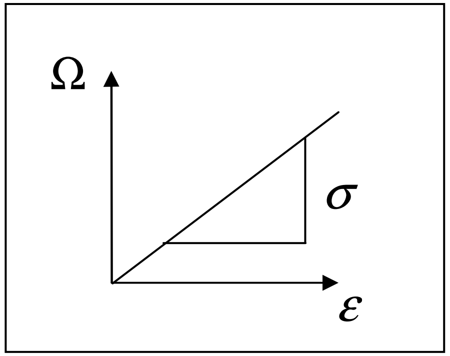
1. total differential $d\Omega(\varepsilon,u)$ $\quad d\Omega(\varepsilon,u)=\frac{\partial\Omega}{\partial\varepsilon}d\varepsilon+\frac{\partial\Omega}{\partial u}du$

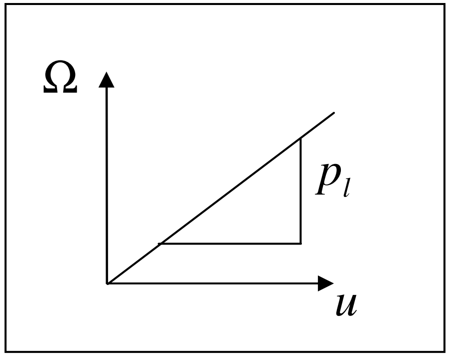


model is isothermal

coupled moisture and mechanical behavior

1. total differential $d\Omega(\varepsilon,u)$	$d\Omega(\varepsilon,u)=\frac{\partial\Omega}{\partial\varepsilon}d\varepsilon+\frac{\partial\Omega}{\partial u}du$			
2. Legendre transformation	$\frac{\partial\Omega}{\partial\varepsilon}\equiv\sigma$	$\frac{\partial\Omega}{\partial u}\equiv p_l$	⟶	$w(\sigma,p_l)=\Omega(\varepsilon,u)-\sigma\varepsilon-p_l u$





coupled moisture and mechanical behavior

1. total differential $d\Omega(\varepsilon,u)$	$d\Omega(\varepsilon,u)=\frac{\partial\Omega}{\partial\varepsilon}d\varepsilon+\frac{\partial\Omega}{\partial u}du$	
2. Legendre transformation	$\frac{\partial\Omega}{\partial\varepsilon}\equiv\sigma \qquad \frac{\partial\Omega}{\partial u}\equiv p_l \longrightarrow$	$w(\sigma,p_l)=\Omega(\varepsilon,u)-\sigma\varepsilon-p_l u$
3. total differential $d\varepsilon, du$	$d\varepsilon=\frac{\partial\varepsilon}{\partial p_l}dp_l+\frac{\partial\varepsilon}{\partial\sigma}d\sigma$	$du=\frac{\partial u}{\partial p_l}dp_l+\frac{\partial u}{\partial\sigma}d\sigma$

coupled moisture and mechanical behavior

1. total differential $d\Omega(\varepsilon,u)$	$d\Omega(\varepsilon,u)=\frac{\partial\Omega}{\partial\varepsilon}d\varepsilon+\frac{\partial\Omega}{\partial u}du$	
2. Legendre transformation	$\frac{\partial\Omega}{\partial\varepsilon}\equiv\sigma \qquad \frac{\partial\Omega}{\partial u}\equiv p_l \longrightarrow$	$w(\sigma,p_l)=\Omega(\varepsilon,u)-\sigma\varepsilon-p_l u$
3. total differential $d\varepsilon, du$	$d\varepsilon=\frac{\partial\varepsilon}{\partial p_l}dp_l+\frac{\partial\varepsilon}{\partial\sigma}d\sigma$	$du=\frac{\partial u}{\partial p_l}dp_l+\frac{\partial u}{\partial\sigma}d\sigma$

strain capacity	moisture capacity	coupling coefficient
$C:=-\frac{\partial^2 w}{\partial\sigma^2}$	$M:=-\frac{\partial^2 w}{\partial p_l^2}$	$B:=-\frac{\partial^2 w}{\partial p_l\,\partial\sigma}$

$$d\varepsilon=Bdp_l+Cd\sigma \qquad du=Mdp_l+Bd\sigma$$

coupled moisture and mechanical behavior

1. total differential $d\Omega(\varepsilon,u)$	$d\Omega(\varepsilon,u)=\frac{\partial\Omega}{\partial\varepsilon}d\varepsilon+\frac{\partial\Omega}{\partial u}du$		
2. Legendre transformation	$\frac{\partial\Omega}{\partial\varepsilon}\equiv\sigma \qquad \frac{\partial\Omega}{\partial u}\equiv p_l$	$\longrightarrow$	$w(\sigma,p_l)=\Omega(\varepsilon,u)-\sigma\varepsilon-p_l u$
3. total differential $d\varepsilon, du$	$d\varepsilon=\frac{\partial\varepsilon}{\partial p_l}dp_l+\frac{\partial\varepsilon}{\partial\sigma}d\sigma$		$du=\frac{\partial u}{\partial p_l}dp_l+\frac{\partial u}{\partial\sigma}d\sigma$

strain capacity	moisture capacity	coupling coefficient
$C_{ijkl}=-\frac{\partial^2 w}{\partial\sigma_{ij}\partial\sigma_{kl}}$	$M=-\frac{\partial^2 w}{\partial p_l^2}$	$B_{ij}=-\frac{\partial^2 w}{\partial p_l\partial\sigma_{ij}}$

$$d\varepsilon_{ij}=C_{ijkl}d\sigma_{kl}+B_{ij}dp_l$$

$$du=Mdp_l+B_{ij}d\sigma_{ij}$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

w_{stress}

$$w_{stress} = \left[\begin{pmatrix} \lambda_{11}^{[1]}\sigma_1^2 + \\ \lambda_{22}^{[1]}\sigma_2^2 + \\ \lambda_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \lambda_{12}^{[1]}\sigma_1\sigma_2 + \\ \lambda_{13}^{[1]}\sigma_1\sigma_3 + \\ \lambda_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \lambda_{4}^{[1]}\sigma_4^2 + \\ \lambda_{5}^{[1]}\sigma_5^2 + \\ \lambda_{6}^{[1]}\sigma_6^2 \end{pmatrix} \right]$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

w_{stress}

$$w_{stress} = \left[\begin{pmatrix} \lambda_{11}^{[1]}\sigma_1^2 + \\ \lambda_{22}^{[1]}\sigma_2^2 + \\ \lambda_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \lambda_{12}^{[1]}\sigma_1\sigma_2 + \\ \lambda_{13}^{[1]}\sigma_1\sigma_3 + \\ \lambda_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \lambda_4^{[1]}\sigma_4^2 + \\ \lambda_5^{[1]}\sigma_5^2 + \\ \lambda_6^{[1]}\sigma_6^2 \end{pmatrix} \right]$$

example: 2'nd order approach

$$w_{stress} = \lambda\sigma^2 \qquad w_{water} = 0 \quad w_{coupling} = 0 \longrightarrow w = \lambda\sigma^2$$

$$C = -\frac{\partial^2 w}{\partial \sigma^2} = -\lambda = const.$$

$$B = -\frac{\partial^2 w}{\partial p_l \partial \sigma} = 0$$

$$d\varepsilon = Bdp_l + Cd\sigma \longrightarrow d\varepsilon = Cd\sigma \longrightarrow \varepsilon = C\sigma$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

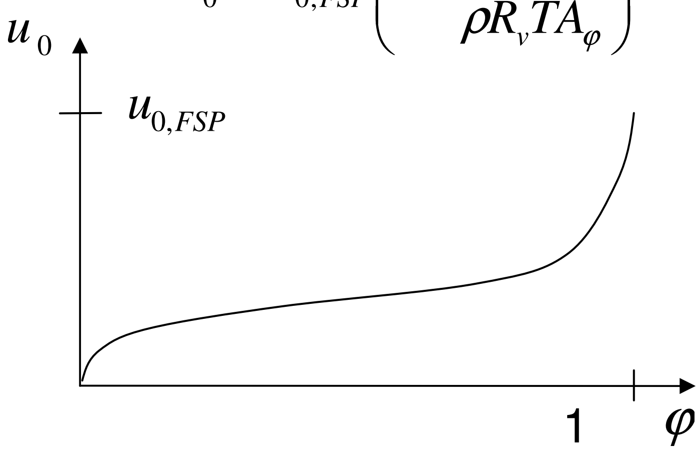
w_{stress}

$$w_{stress} = \left[\begin{pmatrix} \lambda_{11}^{[1]}\sigma_1^2 + \\ \lambda_{22}^{[1]}\sigma_2^2 + \\ \lambda_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \lambda_{12}^{[1]}\sigma_1\sigma_2 + \\ \lambda_{13}^{[1]}\sigma_1\sigma_3 + \\ \lambda_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \lambda_4^{[1]}\sigma_4^2 + \\ \lambda_5^{[1]}\sigma_5^2 + \\ \lambda_6^{[1]}\sigma_6^2 \end{pmatrix}\right]$$

w_{water}

$$w_{water} = -\int u_0(p_l)dp_l$$

$$u_0 = u_{0,FSP}\left(1 - \frac{p_l - p_g}{\rho R_v T A_\varphi}\right)^{-1/n}$$



$$w = \Omega(\varepsilon, u) - \sigma\varepsilon - p_l u$$

$$dw = -udp_l$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

w_{stress}

$$w_{stress} = \left[\begin{pmatrix} \lambda_{11}^{[1]}\sigma_1^2 + \\ \lambda_{22}^{[1]}\sigma_2^2 + \\ \lambda_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \lambda_{12}^{[1]}\sigma_1\sigma_2 + \\ \lambda_{13}^{[1]}\sigma_1\sigma_3 + \\ \lambda_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \lambda_{4}^{[1]}\sigma_4^2 + \\ \lambda_{5}^{[1]}\sigma_5^2 + \\ \lambda_{6}^{[1]}\sigma_6^2 \end{pmatrix} \right]$$

w_{water}

$$w_{water} = -\int u_0(p_l)dp_l = u_{0,FSP}(p_l - \rho R_v T A_\varphi)\left(1 - \frac{p_l}{\rho R_v T A_\varphi}\right)^{-1/n_\varphi} \frac{n_\varphi}{1-n_\varphi}$$

energy function (3D approach)

$$w = w_{stress} + w_{water} + w_{coupling}$$

w_{stress}

$$w_{stress} = \left[\begin{pmatrix} \lambda_{11}^{[1]}\sigma_1^2 + \\ \lambda_{22}^{[1]}\sigma_2^2 + \\ \lambda_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \lambda_{12}^{[1]}\sigma_1\sigma_2 + \\ \lambda_{13}^{[1]}\sigma_1\sigma_3 + \\ \lambda_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \lambda_{4}^{[1]}\sigma_4^2 + \\ \lambda_{5}^{[1]}\sigma_5^2 + \\ \lambda_{6}^{[1]}\sigma_6^2 \end{pmatrix} \right]$$

w_{water}

$$w_{water} = -\int u_0(p_l)dp_l = u_{0,FSP}(p_l - \rho R_v T A_\varphi)\left(1 - \frac{p_l}{\rho R_v T A_\varphi}\right)^{-1/n_\varphi} \frac{n_\varphi}{1 - n_\varphi}$$

$w_{coupling}$

$$u_0(p_l) = u_{0,FSP}\left(1 - \ln\varphi / A_\varphi\right)^{-1/n} \quad \varphi = Exp\left(\frac{p_l - p_g}{\rho_l R_v T}\right)$$

$$w_{coupling} = u_0(p_l)\left[\begin{pmatrix} \gamma_{11}^{[0]}\sigma_1 + \gamma_{11}^{[1]}\sigma_1^2 + \\ \gamma_{22}^{[0]}\sigma_2 + \gamma_{22}^{[1]}\sigma_2^2 + \\ \gamma_{33}^{[0]}\sigma_3 + \gamma_{33}^{[1]}\sigma_3^2 \end{pmatrix} + \begin{pmatrix} \gamma_{12}^{[1]}\sigma_1\sigma_2 + \\ \gamma_{13}^{[1]}\sigma_1\sigma_3 + \\ \gamma_{23}^{[1]}\sigma_2\sigma_3 \end{pmatrix} + \begin{pmatrix} \gamma_{4}^{[0]}\sigma_4 + \gamma_{4}^{[1]}\sigma_4^2 + \\ \gamma_{5}^{[0]}\sigma_5 + \gamma_{5}^{[1]}\sigma_5^2 + \\ \gamma_{6}^{[0]}\sigma_6 + \gamma_{6}^{[1]}\sigma_6^2 \end{pmatrix} \right]$$

conclusion

→ Thermodynamic approach yields set of two constitutive equations linking mechanical and moisture behavior

$$d\varepsilon = Bdp_l + Cd\sigma \qquad du = Mdp_l + Bd\sigma$$

→ Important effects of the coupled moisture and mechanical behavior of wood are covered.

→ Orthotropy of wood is taken into account.

Thank you