COST Action FP0802 Workshop
Thermo-Hygro-Mechanically Modified Wood
Thematic Session in Cooperation with COST Action FP0904
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A computational framework for stress calculation in wood products under humidity and temperature variations

Stefania Fortino
Thermo-Hygro-Mechanical Modelling of wood behaviour and processes
What can we learn from the research on Moisture Induced Stresses?

Background

- **Improved Moisture project** (WoodWisdom-Net, 2007-2010)
  Partners: VTT, TUWien/Austria, KTH/Sweden, MPA/Germany, Harbin University/China, coordination: VTT (Tomi Toratti).
  Topics: computational and experimental wood mechanics, multiphysics and multiscale methods for wood (from micro- to macroscale).

Microscale
Cell wall layer
Unit Cell Method, FEM
(TUW, Austria, Karin Hofstetter’s group)

- Stiffness properties for softwood (spruce and pine) with dependency on density and moisture content
- Hygroexpansion coefficients
- Viscoelastic parameters

Influence of microfibril angle and orientation (based on nano-indentation tests)

Macroscale
Constitutive models
VTT

Unit Cell Method

latewood
earlywood

growth rings

\( \mu = 20-40 \) µm

\( l_sW = 2-4 \) mm
3D Orthotropic-viscoelastic-mechanosorptive constitutive model
Stefania Fortino, Tomi Toratti et al.

Thermomechanics approach
Helmholtz free energy

$$\Psi(T, u, \epsilon, \epsilon^{ve}_{i}, \epsilon^{ms}_{j}, \epsilon^{ms, irr}) = \Phi(u, T) + \Phi^e(\epsilon^e) + \Phi^{ve}(\epsilon^{ve}_{i}) + \Phi^{ms}(\epsilon^{ms}_{j}) + \Phi^{ms, irr}(\epsilon^{ms, irr})$$


- The model is a specialization of the Hanhijärvi and Mackenzie-Helwnein model for drying (2003).
3D Finite Element Modelling for wood

- **3D constitutive model**
  - **wood as orthotropic material**
  - **extension to 3D of previous 1D models**: Toratti (1992), Toratti and Svensson (2002)

- **FEM model**

  Example: Toratti and Svensson’s test (2002), Scots Pine.

- **Validation with respect to experimental data**

  - **elastic strain, hygroexpansion**
  - **viscoelastic creep, mechanosorption**
    (modeled by Kelvin elements)
Model matrices

Viscoelastic matrix

Based on Toratti’s 1D model, 1992

$$C_{^{90}}^{\text{visc}} = J_{^{90}}^{\text{visc}} C_{^{90}}^{\text{visc}}^{-1} \left( t_{\text{ref}}, T_{\text{ref}} \right) = J_{^{90}}^{\text{visc}}$$

$$\begin{pmatrix}
\frac{1}{E^R} & -\frac{\nu^R}{E^T} & 0 & 0 & 0 \\
-\frac{\nu^R}{E^T} & \frac{1}{E^T} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\nu^R E_L} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\nu^L E_L} \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}\left( t_{\text{ref}}, T_{\text{ref}} \right)$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\tau_i^{90}$ [N]</th>
<th>$J^{90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>0.085</td>
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<tr>
<td>2</td>
<td>24</td>
<td>0.035</td>
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<tr>
<td>3</td>
<td>240</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>2400</td>
<td>0.2</td>
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</table>

Matrix for irrecoverable mechanosorption

Based on Toratti and Svensson’s 1D model, 2002

$$\varepsilon^{\text{MS, irr}} = C^{\text{MS, irr}}^{-1} : \sigma \ | \ \dot{U} |$$

$$C^{\text{MS, irr}}^{-1} =$$

$$\begin{pmatrix}
-n_{v}^{E_T} & \nu^{v} & -n_{v}^{E_T} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\nu^{v} & n_{v}^{E_T} & 0 & 0 \\
0 & 0 & 0 & 0 & -\nu^{v} & n_{v}^{G_{FL}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{v}^{E_T} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$m_{v} = 0.006 [\text{MPa}^{-1}]$

Matrix for recoverable mechanosorption

Based on Toratti and Svensson’s 1D model, 2002

$$C_{^{90}}^{\text{rec}} = J_{^{90}}^{\text{rec}} C_{^{90}}^{\text{rec}}^{-1} \left( t_{\text{ref}}, T_{\text{ref}} \right) = J_{^{90}}^{\text{rec}}$$

$$\begin{pmatrix}
\frac{1}{E^R} & -\frac{\nu^R}{E^T} & 0 & 0 & 0 \\
-\frac{\nu^R}{E^T} & \frac{1}{E^T} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\nu^R E_L} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\nu^L E_L} \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}\left( t_{\text{ref}}, T_{\text{ref}} \right)$$

$$J_{^{90}}^{\text{rec}} = 0.7 \left( \frac{1}{E^L (t_{\text{ref}}, T_{\text{ref}})} \right)$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\tau_i^{90}$ [N]</th>
<th>$J_{^{90}}^{\text{rec}}$ [1 MPa]</th>
<th>$J_{^{90}}^{\text{rec}}$ [1 MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>0.0006</td>
<td>0.25 $J_{^{90}}^{\text{rec}}$</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.0006</td>
<td>0.7 $J_{^{90}}^{\text{rec}}$</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>0.0005</td>
<td>0.05 $J_{^{90}}^{\text{rec}}$</td>
</tr>
</tbody>
</table>

- On-going work: new matrices based on recent micromechanics results
- Future work: new matrices for treated wood products
Some implementation details

Update of the creep strains

- Rate equation for a single viscoelastic element:
  \[ \varepsilon_i^{\text{vo}} + \frac{1}{\tau_j} \varepsilon_i^{\text{vo}} = \frac{1}{\tau_j} \varepsilon_i^{\text{vo}-1} : \sigma \]

  \[ \varepsilon_i^{\text{vo},k+1} = \varepsilon_i^{\text{vo}} \exp(-\xi_i) + \left( \frac{C_i^{\text{vo}}}{T_k(\xi_i)} \right)^{-1} : \sigma_k + \left( \frac{C_i^{\text{vo}}}{T_{k+1}(\xi_i)} \right)^{-1} : \sigma_{k+1} \]
  \[ \xi_i = \frac{\Delta t}{\tau_i}; \quad \Delta t = t_{k+1} - t_k; \]
  \[ T_{k+1}(\xi) = 1 - \frac{1}{\xi} [1 - \exp(-\xi)]; \quad T_k(\xi) = 1 - \exp(-\xi) - T_{k+1}(\xi) \]

- Rate equation for a single mechanosorptive element:
  \[ \varepsilon_j^{\text{ms}} = \frac{C_j^{\text{ms}-1}}{\tau_j} : \sigma - \varepsilon_j^{\text{ms}} - |\sigma| \]

- Mechanosorptive strain update: same scheme (but \( \Delta t \) instead of \( \Delta \sigma \) and \( C_j^{\text{ms}} \) instead of \( C^{\text{vo}} \))

**NOTE:** general computational algorithm suitable also for treated wood products

**moisture content history from Fickian or multi-Fickian analysis**

Algorithm implemented into UMAT

- Stress increment at the current time step:
  \[ \Delta \sigma_{k+1} = C_T : (\Delta \varepsilon_{k+1} - \Delta \varepsilon^{\text{vo}} - \Delta \varepsilon_{k+1}^{\text{vo},\text{ms}} + \sum_{i=1}^{m} R_i^{\sigma} + \sum_{j=1}^{n} R_j^{\sigma}) \]
  with \( \Delta \varepsilon_{k+1}^{\text{vo}} = \alpha \Delta \sigma \) and \( \Delta \varepsilon_{k}^{\text{vo}} = \alpha_k \left( 1 - \frac{h}{\alpha_k} \varepsilon_k \right) \Delta \sigma_{k+1} \).

- Tangent operator of the model:
  \[ C_T = \left( C^{\text{vo}} + \sum_{i=1}^{k} \left( \frac{C_i^{\text{vo}}}{T_k(\xi_i)} \right)^{-1} + \sum_{j=1}^{m} \left( \frac{C_j^{\text{ms}}}{T_{k+1}(\xi_j)} \right)^{-1} \right)^{-1} \]

- Viscoelastic and mechanosorptive strain increments:
  \[ \Delta \varepsilon_{k+1}^{\text{vo}} = \left( \frac{C_i^{\text{vo}}}{T_k(\xi_i)} \right)^{-1} : \Delta \sigma_{k+1} - R_i^{\text{vo}}(\varepsilon^{\text{vo}}_{k+1}, \sigma_k); \quad \Delta \varepsilon_{k+1}^{\text{ms}} = \left( \frac{C_j^{\text{ms}}}{T_{k+1}(\xi_j)} \right)^{-1} : \Delta \sigma_{k+1} - R_j^{\text{ms}}(\varepsilon^{\text{ms}}_{k+1}, \sigma_k) \]

- Updated vectors (at iteration step, until \( \| R_{k+1} \| < \text{TOL} \)):
  \[ \sigma_k^{(l+1)} = \sigma_k^{(l)} + \Delta \sigma_{k+1} \]
  \[ \varepsilon_{k+1}^{\text{vo}} = \varepsilon_{k+1}^{\text{vo}} + \Delta \varepsilon_{k+1}^{\text{vo}} \]
  \[ \varepsilon_{k+1}^{\text{ms}} = \varepsilon_{k+1}^{\text{ms}} + \Delta \varepsilon_{k+1}^{\text{ms}} \]
Structural scale - Some applications to civil engineering problems

Glulam beams subjected to wetting or drying in laboratory (Jönsson, 2004)

Timber bridge under natural environmental conditions (city of Lisbon)

Stresses perpendicular to grain after a wetting process

S1 sensor, moisture content in 3 different points (surface, 1 cm depth, 2 cm depth)
Moisture transfer in wood

moisture transfer at the microscale: multi-Fickian approach (water vapor and bound water phases)

macroscale model
Multi-Fickian model by Frandsen et al.

- Multi-Fickian (or multi-phase) model (Frandsen et al., 2007)
  
a) water vapor diffusion in the cell lumens
  \[ c_v = \text{water vapor concentration} \]
  
b) sorption of bound-water (coupling)

  c) bound-water diffusion in the cell walls
  \[ c_b = \text{bound water concentration} \]

\[
\frac{\partial c_v}{\partial t} = \nabla \cdot (D_v \nabla c_v) - \dot{c}
\]

Sorption rate

\[
\frac{\partial c_b}{\partial t} = \nabla \cdot (D_b \nabla c_b) + \dot{c}
\]


Sorption rate and sorption isotherm

- \( H_c \): moisture-dependent reaction rate function
- \( c_0, c_0, \text{ and } m_{bl} \) (density in the reference conditions, \( m_{bl} \) : sorption curve)
- \( h \): relative humidity (%)

\[ \dot{c} = H_c(c_{0d} - c_b) \]

\[ H_c = \begin{cases} 
C_1 \exp \left( -C_2 \left( \frac{c_b}{c_{0d}} \right)^n \right) + C_4 & c_b < c_{0d} \\
C_1 \exp \left( -C_2 \left( 2 - \frac{c_b}{c_{0d}} \right)^n \right) + C_4 & c_b > c_{0d} 
\end{cases} \]

\[ C_2 = c_2 \exp(c_2 h) + c_3 \exp(c_3 h) \]

\[ m_{bl} = \frac{h}{f_1 + f_2 h + f_3 h^2} \]

- State-of-art: new research results on transport properties for wood based on micromechanics research (Eitelberger and Hofstetter, 2011)

- Future work: further research needed for diffusion parameters in treated wood products

**Diffusion coefficient for water vapor** (Schirmer, 1938)

\[ D_v = \xi \left( 2.31 \times 10^{-5} \frac{P_{atm}}{P_{atm} + P_v} \left( \frac{T}{273 K} \right)^{1.81} \right) m^2 s^{-1} \]

- Diffusion coefficient for bound-water (Siar and Siau, 1981)

\[ D_h = D_0^h \exp \left( \frac{-E_a}{RT} \right) m^2 s^{-1} \]

where \( E_a \) is the activation energy for bound-water diffusion (J mol\(^{-1}\)). The diagonal terms of \( D_0 \), i.e., \( D_{xx} \) and \( D_{yy} \) are equal to \( 7 \times 10^{-6} \) m\(^2\) s\(^{-1}\) and \( 17.5 \times 10^{-6} \) m\(^2\) s\(^{-1}\), respectively, for a two-dimensional case (Siau, 1984). The activation energy may be approximated by the linear expression \( E_a = (395.5 - 295a) \times 10^3 \) J mol\(^{-1}\) (Siau, 1995), where \( m = c_h/c_0 \) is the moisture content and \( P_0 \) is the dry density of wood. The decrease in activation energy with moisture content is due to the decrease in bonding energy at the sorption sites.
Some results from implementation of Frandsen’s method in Abaqus

NMR (nuclear magnetic resonance) experiments on small cylindrical samples of Norway spruce (KTH, Sweden)

Comparisons between experimental data and numerical results - Tangential uncoated case

geometry
boundary conditions
- temperature (23°C)
- relative humidity
Fickian vs multi-Fickian models and influence of the sorption curve on the numerical results

Glulam beams: wetting to RH 80% after 4 months at RH 40% (Jönsson, 2005)

Different sorption curves used for Fickian and multi-Fickian simulations
Moisture induced stresses in timber structures exposed to different climates

- The variations of MC in a point P of a timber member during its service life depend on:
  - wood species
  - environmental variations of relative humidity RH=RH(t);
  - environmental variations of temperature T=T(t)
  - initial moisture content
  - type of exposure
  - size of the timber cross-section
  - location of the point P in the cross-section
  - application of coating on the surface of the member
Applications of the VTT model for wood
Timber structures under European environmental conditions

Identification of climatic regions

The cross-sections of technical interest

- **NARROW:**
  - sawn timber
  - 38 x 225 mm

- **MEDIUM:**
  - glulam made from 12 laminations
  - 115 x 540 mm

- **CONTINUOUS SLAB:**
  - crosslam panel made from 5 laminations
  - 2400 x 150 mm

Note: spruce wood is considered in the analyses

Narrow and medium section - Influence of daily variation of RH on moisture content
(4 measurements in 1 day, Fickian simulations, no hysteresis implementation)

**Uncoated Narrow Section**
- $\Delta RH = 0.45$
- $\Delta MC$ maximum = 0.5
- High gradient

**Coated Narrow Section**
- $\Delta RH = 0.45$
- $\Delta MC$ maximum = 0.05
- Low gradient

**Uncoated Medium Glulam Section**
- $\Delta RH = 0.45$
- $\Delta MC$ maximum = 0.5
- High gradient

![Graphs showing moisture content changes over distance](image)
Moisture induced stresses in different points of the medium glulam section
1 year analysis – Cities of Lisbon, Rovaniemi and Warsaw

<table>
<thead>
<tr>
<th>Property</th>
<th>Lisbon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension strength parallel to grain</td>
<td>16.5 MPa</td>
</tr>
<tr>
<td>Tension strength perpendicular to grain</td>
<td>0.4 MPa</td>
</tr>
<tr>
<td>Compression strength parallel to grain</td>
<td>24 MPa</td>
</tr>
<tr>
<td>Compression strength perpendicular to grain</td>
<td>2.7 MPa</td>
</tr>
<tr>
<td>Shear strength</td>
<td>2.7 MPa</td>
</tr>
</tbody>
</table>

Eurocode values

- Middle height
- 10 mm
- 1 = Surface point
- 2 = ΔL point
- 3 = Middle point

Graph showing stress distribution over time for different locations.
THM wood processes and products
What can we learn from the research on Moisture Induced Stresses?

During the high temperature heat treatment of wood, it is important to know how the temperature and moisture distribution change with time. This information can be used:

- to adjust the treatment parameters
- to control the quality of final product more effectively.

Effects to be studied

- the influence of initial moisture content
- the influence of the heating rate
- the influence of initial density
- the maximum treatment temperature
- other effects

• Importance of the Hygro-Thermal modelling!

• Suggestions from existing literature on High-Temperature Heat-Treatment of Wood : Research performed by the Group on the Thermotransformation of Wood (GRTB), Canada from 2006 to present (Kokaefe et al.). Mainly Canadian hardwood was studied.
Future work: Thermo-Hygro-Mechanical modelling at high temperatures
max T=120-125 °C

Mechanosorptive-plastic element (including isotropic and kinematic hardening)

Future work: Thermo-Hygro-Chemo-Mechanical modelling at high temperatures

- Models to be built in a thermodynamics framework:

\[
\Psi(T, u, \varepsilon, \varepsilon^v, \varepsilon^m, \varepsilon^{ms, irr}) = \Phi(u, T) + \Phi^c(\varepsilon^c) + \text{contribution of molar volume fractions } \xi_i
\]

Thank You!