Numerical simulation of mechanically and moisture loaded wooden structures

Susanne Saft, Michael Kaliske

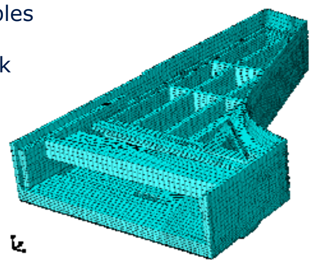
Institute for Structural Analysis
www.tu-dresden.de/isd

KULTURSTIFTUNG DES BUNDES
KULTUR STIFTUNG · DER LÄNDER
Federal Foundation of Culture, Germany

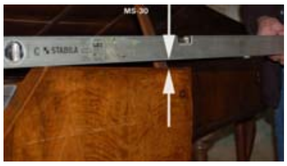
MUSEUM
Händel-Haus Halle, Germany

Outline

- motivation
- modeling of mechanical behavior
- modeling of moisture transfer
- hygro-mechanical coupling
- examples
- outlook



Motivation



restoration of historical music instruments

- conservation of original substance
- maintenance of original usage

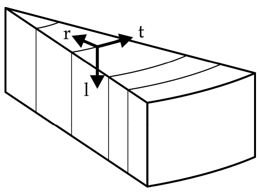
large deformations of the structure, local damages

- caused by string loading
- enforced by climatic fluctuations

analysis of the load bearing behavior of historical keyboard instruments using the Finite Element Method

→ requirement: appropriate material models for simulation of mechanics, moisture transfer and coupling

Modeling of Cylindrical Anisotropy



potential at material coordinate system

$$\Psi = \frac{1}{2}\, \underline{\varepsilon} : \underline{\underline{C}} : \underline{\varepsilon}$$

linear-elastic stress-strain-dependency

$$\underline{\sigma} = \underline{\underline{C}} : \underline{\varepsilon}$$

elasticity tensor in material coordinates

$$\underline{\underline{S}} = \underline{\underline{C}}^{-\mathbf{1}} = \begin{bmatrix} \frac{1}{E_r} & -\frac{v_{rt}}{E_t} & -\frac{v_{rl}}{E_l} & 0 & 0 & 0 \\ -\frac{v_{rt}}{E_t} & \frac{1}{E_t} & -\frac{v_{tl}}{E_l} & 0 & 0 & 0 \\ -\frac{v_{rl}}{E_l} & -\frac{v_{tl}}{E_l} & \frac{1}{E_l} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{rt}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{tl}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{rl}} \end{bmatrix}$$

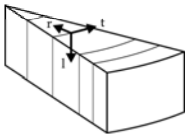
with respect to

$$\frac{E_i}{E_j} = \frac{v_{ji}}{v_{ij}}$$

Schmidt, Kaliske [2009]

Modeling of Cylindrical Anisotropy

rotation between material coordinates and global coordinates



- rotation of global strains to material coordinates

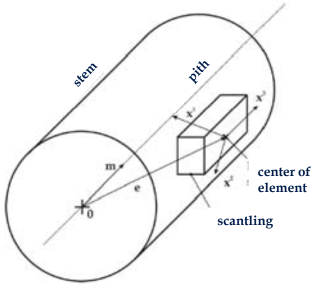
$$^{t+\Delta t}\varepsilon_{kl}^{n+1} = M_{ik} \cdot M_{jl} \cdot {}^{t+\Delta t}\tilde{\varepsilon}_{ij}^{n+1}$$

- rotation of local stresses to global coordinate system

$$^{t+\Delta t}\tilde{\sigma}_{ij}^{n+1} = M_{ki} \cdot M_{lj} \cdot {}^{t+\Delta t}\sigma_{kl}^{n+1}$$

- structural tensor

$$\boldsymbol{M} = \begin{pmatrix} \frac{x_1^1}{|x^1|} & \frac{x_2^1}{|x^1|} & \frac{x_3^1}{|x^1|} \\ \frac{x_1^2}{|x^2|} & \frac{x_2^2}{|x^2|} & \frac{x_3^2}{|x^2|} \\ \frac{x_1^3}{|x^3|} & \frac{x_2^3}{|x^3|} & \frac{x_3^3}{|x^3|} \end{pmatrix}$$



Modeling of Compression Failure

multi-surface plasticity model with C_1-continous yield criterion

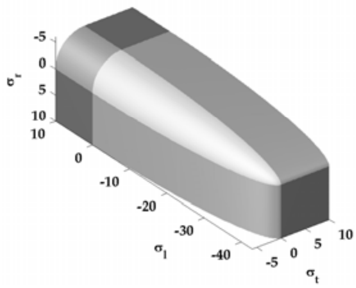
- failure due to compression loading
- ductile, nearly plastic behavior
- C_1-continuous anisotropic yield criterion

$$f = \underline{\sigma} : \underline{\underline{b}} : \underline{\sigma} + q - 1 \leq 0$$

- associated flow rule

$$\dot{\underline{\varepsilon}}^p = \gamma \frac{\partial f(\underline{\sigma}, q)}{\partial \underline{\sigma}}$$

- assumption of linear hardening



Resch, Kaliske [2010]

Modeling of Compression Failure

definition of the C_1-continuous yield criterion

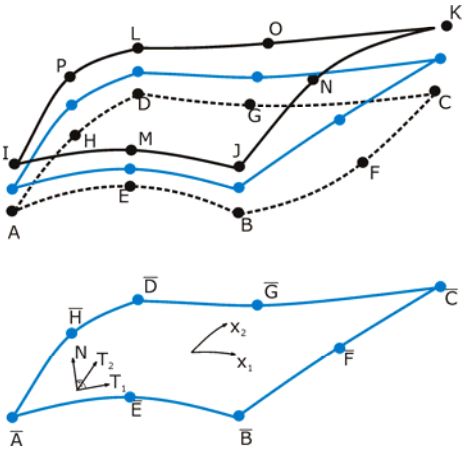
- yield criterion: $f = \underline{\boldsymbol{\sigma}} : \underline{\underline{\boldsymbol{b}}} : \underline{\boldsymbol{\sigma}} + q - 1 \leq 0$
- tensor of strength: $\underline{\underline{\boldsymbol{b}}} = \underline{\boldsymbol{b}}_s \otimes \underline{\boldsymbol{b}}_f$
- activation of octant of stress domain: $\underline{\boldsymbol{b}}_s = \frac{1}{2} \cdot \underline{\boldsymbol{1}} - \frac{1}{2} \cdot sgn\,(\underline{\boldsymbol{\sigma}} \circ \underline{\boldsymbol{1}})$
- strength information: $\underline{\boldsymbol{b}}_f = \begin{bmatrix} \frac{1}{f_{cr}^2} & 0 & 0 \\ 0 & \frac{1}{f_{ct}^2} & 0 \\ 0 & 0 & \frac{1}{f_{cl}^2} \end{bmatrix}$

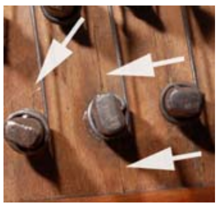
Resch, Kaliske [2010]

TECHNISCHE UNIVERSITÄT DRESDEN
Modeling of Mechanical Behavior

Modeling of Brittle Failure

interface-element formulation



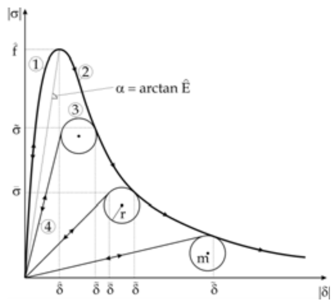


Schmidt, Kaliske [2007]

Modeling of Brittle Failure

interface-material approach

- direction dependent strength values f and fracture energies G
- coupling of material directions
- C_1-continuous material formulation

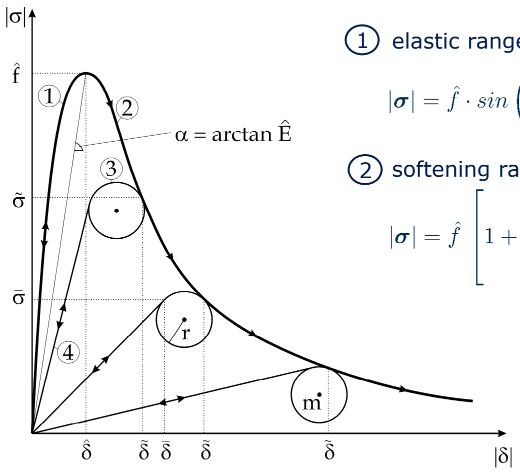


parameters of the model

- displacement at integration point $\boldsymbol{\delta} = [\delta_1\ \delta_2\ \delta_3]^T$
- elastic range $\hat{\delta}$
- parameter of C_1-continuous damage path r
- history variable $h = \dfrac{max(|\boldsymbol{\delta}|)}{\hat{\delta}}$

Schmidt, Kaliske [2007]

Modeling of Brittle Failure



① elastic range

$$|\boldsymbol{\sigma}| = \hat{f} \cdot sin\left(\frac{\pi \cdot |\boldsymbol{\delta}|}{2 \cdot \hat{\delta}}\right)$$

② softening range

$$|\boldsymbol{\sigma}| = \hat{f}\left[1 + \left(\frac{\pi^2\,\hat{f}}{2\left(\pi\,\hat{G} - 2\,\hat{f}\,\hat{\delta}\right)}\right)^2 \left(|\boldsymbol{\delta}| - \hat{\delta}\right)^2\right]^{-1}$$

③ C_1-continuous transition

④ elastic range of damage

Schmidt, Kaliske [2007]

Modeling of Moisture Diffusion

moisture transport below fibre saturation point (diffusion) described by Fick's Law

- steady state form

$$q = -D \cdot \rho_0 \cdot \nabla m$$

- transient form

$$\frac{\partial m}{\partial t} = \nabla \cdot (D \cdot \nabla m)$$

- q: moisture flux [kg/(mm²s)]
- ρ_0: density in absolute dry condition [kg/mm³]
- m: moisture content [-]
- D: diffusion coefficient subject to moisture content [mm²/s]

Modeling of Moisture Diffusion

anisotropic tensor of moisture conductivity

$$\underline{D} = \begin{bmatrix} D_r(m) & 0 & 0 \\ 0 & D_t(m) & 0 \\ 0 & 0 & D_l(m) \end{bmatrix}$$

- approach of *Hanhijärvi*

$$D_i(m) = a_i \cdot e^{b_i \cdot m}$$

$$a_r = a_t = 8 \cdot 10^{-5} \frac{mm}{s} \qquad a_l = 2 \cdot 10^{-4} \frac{mm}{s} \qquad b_r = b_t = b_l = 4$$

- nonlinearity of D(m) is considered within tangent matrix
→ consistent linearization

$$\frac{\partial D}{\partial m} \neq const$$

Hanhijärvi [1997]

Modeling of Moisture Diffusion

Why Fick's Law?

- transient form of Fick's law merely approximation for simulation of moisture diffusion in wood
- resulting difference between model and experimental results often denoted as "Non-Fick'ian Behavior"
  → difference increases with increasing gradient of moisture content
- more appropriate models require many input parameters, hardly known for most wood types
- application on music instruments: most surfaces varnished

  → increase of surface resistance, decrease of moisture gradient

  → usage of Fick's Law, when diffusions coefficients determined by steady- state experiments

Modeling of Surface Resistance

consideration of changes in ambient atmosphere:
boundary conditions of *Neumann* type

$$q_n = S \cdot \rho_0 \cdot (m_{ecm} - m_s)$$

- q_n: moisture flux across the boundary [kg/(mm²s)]
- ρ_0: density in absolute dry condition [kg/mm³]
- m_{ecm}: equilibrium moisture content [-]
- m_s: moisture content at the surface
- S: surface emissivity coefficient subject to moisture content [mm/s]

Modeling of Surface Resistance

- evaluation of equilibrium moisture content: approach of *Avramidis*

$$m_{ecm} = 0.01 \cdot \left[\frac{-T \cdot ln(1-RH)}{0.13 \cdot (1 - \frac{T}{647.1})^{-6.46}} \right]^{\frac{1}{110 \cdot T^{-0.75}}}$$

- surface emissivity coefficient: approach of *Hanhijärvi*

$$S(m) = a \cdot e^{b \cdot m}$$

$$a = 3.2 \cdot 10^{-5} \frac{mm}{s} \qquad b = 4$$

- nonlinearity of S(m) is considered within tangent matrix
  → consistent linearization

$$\frac{\partial S}{\partial m} \neq const$$

Avramidis [1989], Hanhijärvi [1997]

Coupling of Mechanics and Moisture Diffusion

coupling of anisotropic mechanics and anisotropic moisture transfer

- modeling of influence of moisture content on mechanical properties
- influence of loading state on moisture transport neglectable for small strains
- internal forces of solid-elements: mechanics extended by swelling/shrinkage

$$F = F_m + F_s$$

$$F = \int_V \left(\underline{\boldsymbol{B}}^T : \underline{\boldsymbol{\sigma}}(m,u) \right) dV$$

for linear elasticity

$$F = \int_V \left(\underline{\boldsymbol{B}}^T : \underline{\underline{\boldsymbol{C}}}(m) : \underline{\boldsymbol{\epsilon}}(u) \right) dV - \int_V \left(\underline{\boldsymbol{B}}^T : \underline{\underline{\boldsymbol{C}}}(m) : \underline{\boldsymbol{\epsilon}}(m) \right) dV$$

Coupling of Mechanics and Moisture Diffusion

- vector of shrinkage strains (stress-independent)

$$\underline{\epsilon}(m) = \underline{\beta} \cdot (m - m_{ref})$$

$$\underline{\beta} = \begin{bmatrix} \beta_r & 0 & 0 \\ 0 & \beta_t & 0 \\ 0 & 0 & \beta_l \end{bmatrix}$$

β: shrinkage values [-]

m: nodal moisture content [-]

m_{ref}: reference moisture content [-]

- components of the tangent matrix for linear elastic range

$$K = \frac{\partial F}{\partial m} = \int_V \underline{B}^T : \left(\frac{\partial \underline{\underline{C}}(m)}{\partial m} : (\underline{\epsilon}(u) - \underline{\epsilon}(m)) - \underline{\underline{C}}(m) : \frac{\partial \underline{\epsilon}(m)}{\partial m} \right) dV$$

Moisture-dependent Elastic Properties

components of compliance tensor subject to moisture content

$$\underline{\underline{S}} = \underline{\underline{C}}^{-1} = \begin{bmatrix} S_{11}(m) & S_{12}(m) & S_{13}(m) & 0 & 0 & 0 \\ S_{21}(m) & S_{22}(m) & S_{23}(m) & 0 & 0 & 0 \\ S_{31}(m) & S_{32}(m) & S_{33}(m) & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}(m) & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55}(m) & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66}(m) \end{bmatrix}$$

approximation of experimental results carried out by *Neuhaus* for spruce

$$S_{ij}(m) = a_{ij} \cdot sin(b_{ij} \cdot m + c_{ij}) + d_{ij}$$

$$0.01 \leq m \leq 0.28$$

symmetry of elasticity tensor only partly validated by experimental results
→ interpolation

$$\overline{S}_{ij}(m) = \frac{1}{2} \cdot (S_{ij}(m) + S_{ji}(m))$$

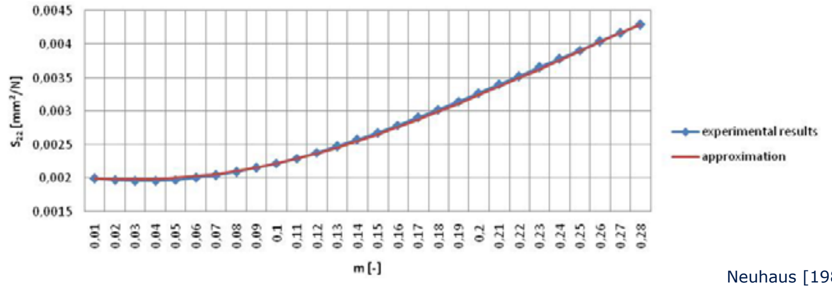
Neuhaus [1981]

Moisture-dependent Elastic Properties

example: $S_{22}(m)$

$$S_{22}(m) = \frac{1}{550} \cdot sin(7.38 \cdot m - 1.79) + 0.0038$$

$$a_{22} = \frac{1}{550}\frac{mm^2}{N} \qquad b_{22} = 7.38[-] \qquad c_{22} = -1.79[-] \qquad d_{22} = 0.0038\frac{mm^2}{N}$$



Neuhaus [1981]

Moisture-dependent Strength Properties

first simplifying approach:

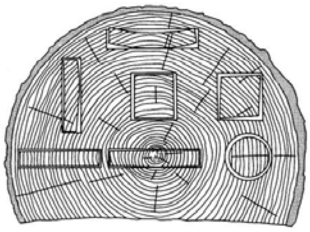
- linear dependence of each strength value on moisture content
- strength decreases with increasing moisture content
- approximate values from literature (mostly spruce)
- validity: $0.06 \leq m \leq 0.3$
- example: compression strength in radial direction

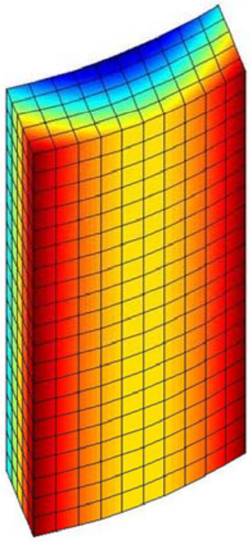
$$f_{c,r}(m) = f_{c,r,12} \cdot \left(1 + \frac{(-4.5 \cdot m + 54)}{100}\right)$$

Gerhards [1982], Kollmann [1982]

Elasticity: Shrinkage Deformations

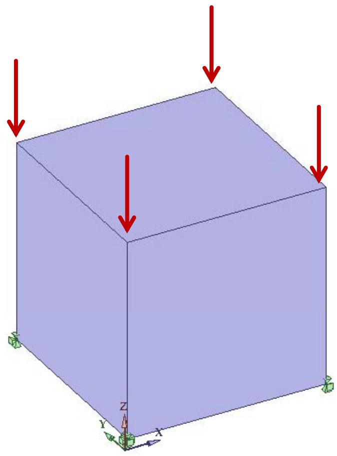
- FE-model
  - material: linear elastic anisotropic mechanics and anisotropic moisture
  - consideration of cylindrical anisotropy
  - steady-state simulation
- loading
  - reduction of moisture content
- realistic simulation of shrinkage deformations



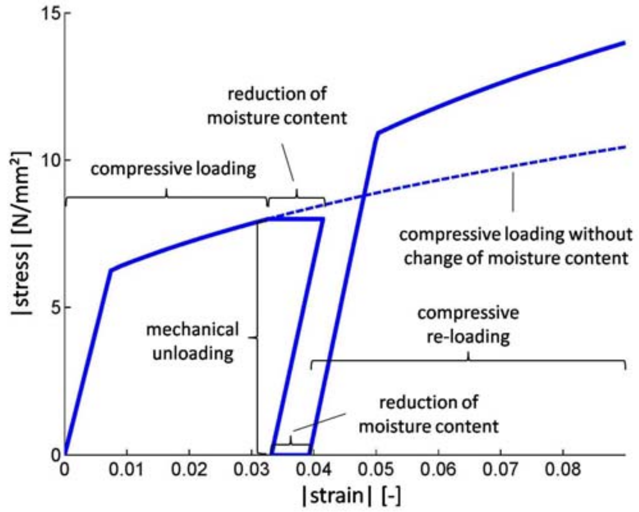


Plasticity: Model Problem

- FE-model
  - element with unit volume and statically determinate support
- loading: 5 load steps
  1. compressive load in radial direction (z-direction)
  2. reduction of moisture content
  3. mechanical unloading
  4. reduction of moisture content
  5. compressive re-loading

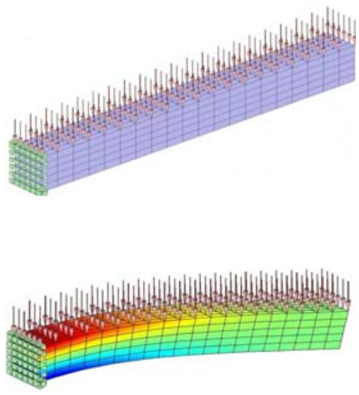


Plasticity: Model Problem

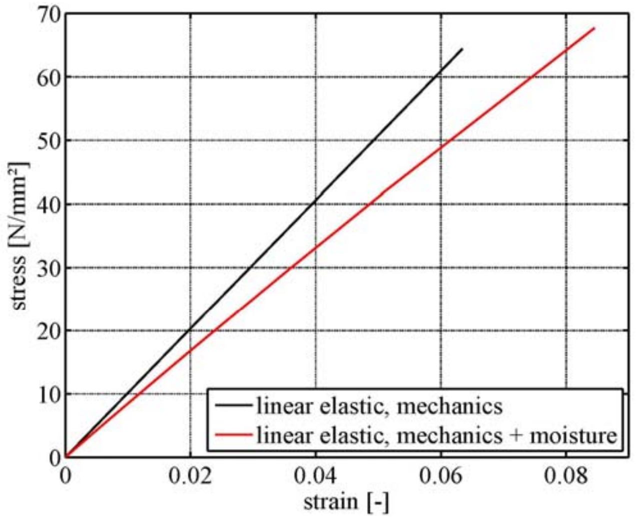


Cantilever Beam

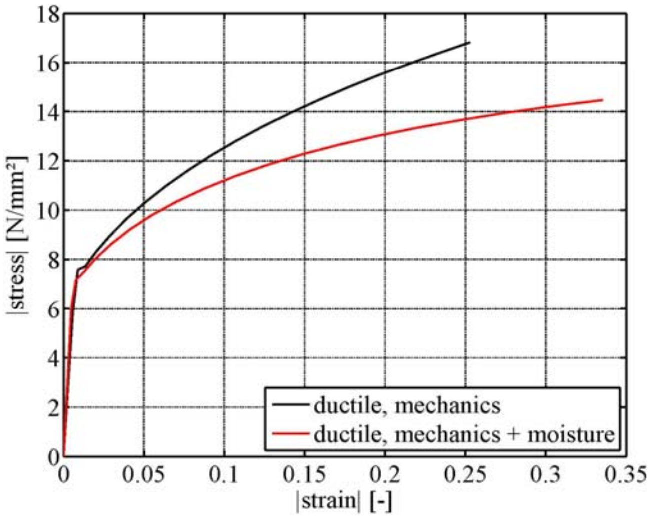
- FE-model
  - material 1: linear elastic anisotropic mechanics and anisotropic moisture
  - material 2: multi-surface plasticity and anisotropic moisture
  - steady-state simulation
- loading
  - constant surface loading
  - change of ambient atmosphere from T_1=10°C, RH_1=0.4 to T_2=28°C, RH_2=0.7



Cantilever Beam

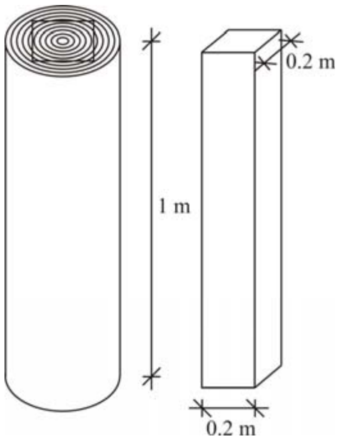


Cantilever Beam



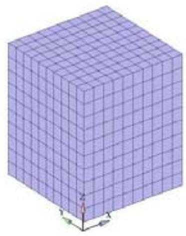
Swelling Beam

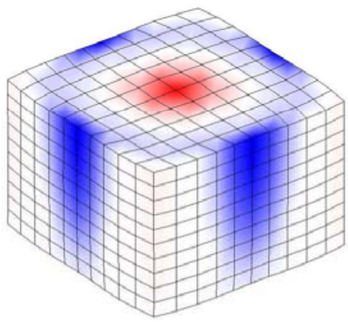
- FE-model
  - beam, virtually cut out of the middle of a stem
  - consideration of cylindrical anisotropy
  - material 1: linear elastic anisotropic mechanics and anisotropic moisture
  - material 2: multi-surface plasticity and anisotropic moisture
  - steady-state and transient simulation



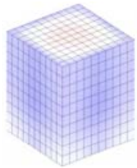
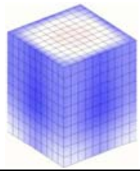
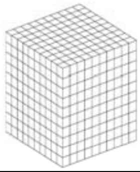
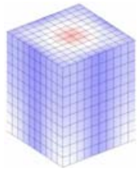
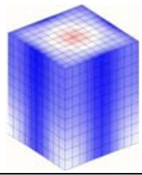
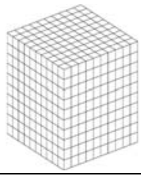
Swelling Beam

- loading
  - increase of moisture content from absolutely dry state to equilibrium moisture content 0.237 (corresponding to ambient climate of T=22°C, RH=0.95)
  - transient simulation: increase of moisture content during 1 h, afterwards simulation continued for further 1.000.000 sec (11.57 days)
- study of lower part of beam

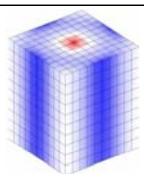
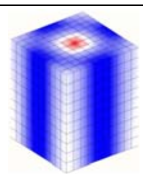
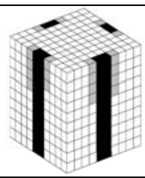
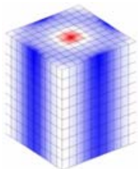
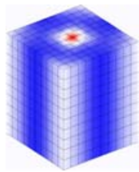
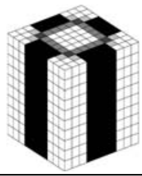




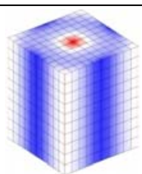
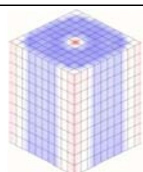
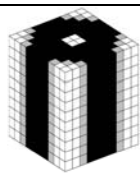
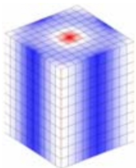
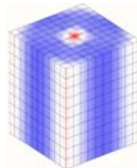
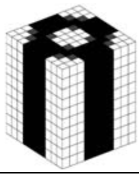
Swelling Beam

	linear elastic 3rd principal stress [N/mm²]	ductile 3rd principal stress [N/mm²]	ductile plastified parts
time	-14,5 -10 -5 0 5 9,5	-6 -4 -2 0 2 4 6 8	
1 min			
10 min			

Swelling Beam

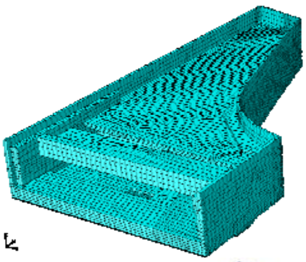
	linear elastic 3rd principal stress [N/mm²]	ductile 3rd principal stress [N/mm²]	ductile plastified parts
time	-14,5 -10 -5 0 5 9,5	-6 -4 -2 0 2 4 6 8	
30 min			
50 min			

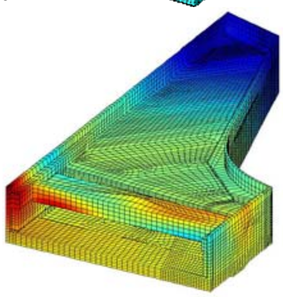
Swelling Beam

	linear elastic 3rd principal stress [N/mm²]	ductile 3rd principal stress [N/mm²]	ductile plastified parts
time	-14,5 -10 -5 0 5 9,5	-6 -4 -2 0 2 4 6 8	
11,62 days			
steady state			

Outlook

- modeling of brittle failure subject to moisture content (fracture mechanics)
- consideration of time-dependent behavior (viscoelastic and mechano-sorptive creep)
- adaption of the material models to other wood types, used in instrument making
- if necessary, development and realisation of experimental studies to evaluate moisture-dependent mechanical properties
- completion, verification and iterative improvement of the FE models of historical pianofortes





TECHNISCHE UNIVERSITÄT DRESDEN

Thank you for your attention.

This project is supported by the KUR-program for the Conservation of Moveable Cultural Assets.

KULTURSTIFTUNG DES BUNDES

KULTUR STIFTUNG · DER LÄNDER

project partners: Händel-Haus Halle

Clemens Birnbaum (director)
Stefan Ehricht (conservator)
Achim Haufe (conservator)